MCI 28.20

MARINE CORPS INSTITUTE

ELECTRONICS MATHEMATICS FOR MARINES

MARINE BARRACKS
WASHINGTON, DC

ELECTRONICS MATHEMATICS FOR MARINES

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Student Information

Number and Title	MCI 2820 ELECTRONICS MATHEMATICS FOR MARINES
Study Hours	32
Course Materials	Text
Review Agency	Marine Corps Communications Electronics School MCCES
Reserve Retirement Credits (RRC)	11
ACE	Course submitted for review by the American Council on Education.
Assistance	For administrative assistance, have your training officer or NCO log on to the MCI home page at www.mci.usmc.mil. Marines CONUS may call toll free 1-800-MCI-USMC. Marines worldwide may call commercial (202) 685-7596 or DSN 325-7596.

Study Guide

Congratulations

Congratulations on your enrollment in a distance education course from the Distance Learning and Technologies Department (DLTD) of the Marine Corps Institute (MCI). Since 1920, the Marine Corps Institute has been helping tens of thousands of hard-charging Marines, like you, improve their technical job performance skills through distance learning. By enrolling in this course, you have shown a desire to improve the skills you have and master new skills to enhance your job performance. The distance learning course you have chosen, MCI 2820, *Electronics Mathematics for Marines*, provides Marines with the foundation and basic knowledge of algebraic and determinant mathematics to support them in this changing technological age. The intent of this course is not to qualify you in any occupational field, but to provide you with the basic knowledge in advanced mathematics in support of your mission.

Your Personal Characteristics

- **YOU ARE PROPERLY MOTIVATED.** You have made a positive decision to get training on your own. Self-motivation is perhaps the most important force in learning or achieving anything. Doing whatever is necessary to learn is motivation. You have it!

- **YOU SEEK TO IMPROVE YOURSELF.** You are enrolled to improve those skills you already possess, and to learn new skills. When you improve yourself, you improve the Corps!

- **YOU HAVE THE INITIATIVE TO ACT.** By acting on your own, you have shown you are a self-starter, willing to reach out for opportunities to learn and grow.

- **YOU ACCEPT CHALLENGES.** You have self-confidence and believe in your ability to acquire knowledge and skills. You have the self-confidence to set goals and the ability to achieve them, enabling you to meet every challenge.

- **YOU ARE ABLE TO SET AND ACCOMPLISH PRACTICAL GOALS.** You are willing to commit time, effort, and the resources necessary to set and accomplish your goals. These professional traits will help you successfully complete this distance learning course.

Continued on next page

Beginning Your Course

Before you actually begin this course of study, read the student information page. If you find any course materials missing, notify your training officer or training NCO. If you have all the required materials, you are ready to begin.

To begin your course of study, familiarize yourself with the structure of the course text. One way to do this is to read the table of contents. Notice the table of contents covers specific areas of study and the order in which they are presented. You will find the text divided into several study units. Each study unit is comprised of two or more lessons and lesson exercises.

Leafing Through the Text

Leaf through the text and look at the course. Read a few lesson exercise questions to get an idea of the type of material in the course. If the course has additional study aids, such as a handbook or plotting board, familiarize yourself with them.

The First Study Unit

Turn to the first page of study unit 1. On this page, you will find an introduction to the study unit and generally the first study unit lesson. Study unit lessons contain learning objectives, lesson text, and exercises.

Reading the Learning Objectives

Learning objectives describe in concise terms what the successful learner, you, will be able to do as a result of mastering the content of the lesson text. Read the objectives for each lesson and then read the lesson text. As you read the lesson text, make notes on the points you feel are important.

Completing the Exercises

To determine your mastery of the learning objectives and text, complete the exercises developed for you. Exercises are located at the end of each lesson, and at the end of each study unit. Without referring to the text, complete the exercise questions and then check your responses against those provided.

Continued on next page

Continuing to March

Continue on to the next lesson, repeating the above process until you have completed all lessons in the study unit. Follow the same procedures for each study unit in the course.

Preparing for the Final Exam

To prepare for your final exam, you must review what you learned in the course. The following suggestions will help make the review interesting and challenging.

- **CHALLENGE YOURSELF.** Try to recall the entire learning sequence without referring to the text. Can you do it? Now look back at the text to see if you have left anything out. This review should be interesting. Undoubtedly, you'll find you were not able to recall everything. But with a little effort, you'll be able to recall a great deal of the information.

- **USE UNUSED MINUTES.** Use your spare moments to review. Read your notes or a part of a study unit, rework exercise items, review again; you can do many of these things during the unused minutes of every day.

- **APPLY WHAT YOU HAVE LEARNED.** It is always best to use the skill or knowledge you've learned as soon as possible. If it isn't possible to actually use the skill or knowledge, at least try to imagine a situation in which you would apply this learning. For example make up and solve your own problems. Or, better still, make up and solve problems that use most of the elements of a study unit.

- **USE THE "SHAKEDOWN CRUISE" TECHNIQUE.** Ask another Marine to lend a hand by asking you questions about the course. Choose a particular study unit and let your buddy "fire away." This technique can be interesting and challenging for both of you!

- **MAKE REVIEWS FUN AND BENEFICIAL.** Reviews are good habits that enhance learning. They don't have to be long and tedious. In fact, some learners find short reviews conducted more often prove more beneficial.

Continued on next page

Tackling the Final Exam

When you have completed your study of the course material and are confident with the results attained on your study unit exercises, take the sealed envelope marked **"FINAL EXAM"** to your unit training NCO or training officer. Your training NCO or officer will administer the final examination and return the examination and the answer sheet to MCI for grading. Before taking your final examination, read the directions on the DP-37 answer sheet carefully.

Completing Your Course

The sooner you complete your course, the sooner you can better yourself by applying what you've learned! HOWEVER--you do have 2 years from the date of enrollment to complete this course.

Graduating!

As a graduate of this distance education course and as a dedicated Marine, your job performance skills will improve, benefiting you, your unit, and the Marine Corps.

Semper Fidelis!

STUDY UNIT 1

INTRODUCTION TO ELECTRONICS MATHEMATICS FOR MARINES

Introduction. The successful accomplishment of almost any assigned mission will rely on Marines who are assigned to military occupational specialties (MOSs) where knowledge of mathematics is essential. You must know how, when, and where to apply the mathematical theories that you will learn to actual situations and mission assignments. This study unit addresses such concerns. This study unit provides basic foundation information. For many of you it may be just a review. However, it is important for you to complete the study unit as if you were a beginner. You will learn mathematical terms and their meanings. When you complete the lesson, you should be able to apply those terms in situations when required.

In addition to learning mathematical terms and how to apply them, you will be able to solve algebraic problems by applying the rules of addition, subtraction, multiplication, and division. You will also learn to apply basic rules in solving monomials and polynomials.

Lesson 1. MATHEMATICAL TERMS FOR USE

LEARNING OBJECTIVE

Define algebraic terms without the aid of references.

1101. Interpret Mission Assignment

You may be assigned to a unit, platoon, or division where you may receive a written mission assignment where mathematical terms are used. You must know those terms so that you can decipher the assignment and carry it out according to specifications. Therefore, you must know all terms that are relative to any possible assignment. The first terms that you must know are the basic arithmetic signs of operation. These four signs are the plus$(+)$, minus$(-)$, times$(\bullet)$, and division$(/)$. There are also other symbols that will direct you, the Marine, to perform other required operations. Those symbols are not being ignored; for this lesson the selected four are used as examples.

Examples: The signs of operation in algebra are

$$(+),\ (-),\ (\bullet), \text{ and } (/)$$

The same meanings apply in algebra as in arithmetic. The multiplication sign $(\bullet, *, \times)$ is generally omitted between literal numbers.

Now that we have identified the basic signs of operations, try this easy challenge.

(Multiple Choice)

Which of the following sets of mathematical signs of operations have the same meaning in algebra as in arithmetic?

a. $*, +, \pi$
b. $\times, *, -\pi, +$
c. $-, +, \bullet, /$
d. $-, \bullet, 1, +, \pi$

The correct answer is "c." If you answered incorrectly, review paragraph 1101. If your answer is correct, continue.

Remember, do not fix in your mind that these symbols or signs are the only ones used that request particular operations. Now, focus your attention on numerical expressions, literal expressions, and terms. These definitions and examples will help you as you prepare to use them in an operation.

a. Numerical expressions consist entirely of numbers separated by signs of operations. See the example.

Example: 2+3-5*2

b. Literal expressions are slightly different. They consist of general numbers and letters that are separated by signs of operations. See the example.

Example: $2a+3c-4$

c. A term is an expression that contains literal and/or number parts that are **not** separated by an operation sign such as a plus or a minus. See the example.

Example: $2a$, $3c$, and -4

d. Positive and negative terms are two basic types of terms. A positive term is not always marked with the operative sign $(+)$ at the first term. In fact, the (+) is almost never used. See the example.

Example: $3a^2bc$

Note: For our purposes, a positive $(+)$ sign will never precede the first number or letter of an equation. It is understood that if there is no sign, the operand or sign is considered to be positive.

A negative term always precedes the negative $(-)$ sign. See the example.

Example: $-3a^2bc$

e. Like/similar and unlike/dissimilar terms are two more words you must know the meaning of. Like/similar terms are those terms that have the same bases (numbers and letters) raised to the same power. See the examples.

Examples: $3a^2bc$ $4a^2bc$

Unlike/dissimilar terms are those terms that have the same bases (numerals and letters) raised to different powers. See the examples.

Examples: $3a^2bc$ $4ab^3c$

Try these challenges to reinforce what we have discussed thus far.

(Define)

Define the term "numerical expression."

____

____

____

(Define)

Define the term "literal expression."

____

____

____

(Multiple Choice)

Which of the following signs must always precede the term of an expression?

a. (+)
b. (-)

Check your responses against the following answers to make sure that your responses are correct. If you defined numerical expression as an expression consisting entirely of numbers separated by signs, you are correct. If you defined a literal expression as an expression consisting of general numbers and letters, you are correct. If you answered the third challenge as "b," you are correct. If you answered any of the challenges incorrectly, review paragraph 1101 before continuing. If you answered all challenges correctly, you may proceed to the next challenge.

(Multiple Choice)

Which of the following expressions or groups of expressions is an example of like/similar terms?

a. $2a^2$
b. $2abc$ $2a^2bc$
c. $3a^4bc$ $5a^4bc$
d. $2a$ $4a^4$

(Multiple Choice)

Select the expressions or groups of expressions that are an example of unlike/dissimilar terms.

a. $3ab$
b. $3ab^2$ $3ab^2$
c. $3ab^2$ $4ab^2$
d. $3ab^2c$ $4ab^3c$

The correct responses are for like/similar terms, item "c," and for unlike/dissimilar terms, item "d." If your answers are different, read paragraph 1101 again. If your answers are correct, you may continue.

Now focus your attention on the various types of numerical terms so that you will get a definite understanding of their meanings. You will need to know and use them in future operations. They are monomial, polynomial/multinomial, (including binomial, and trinomial). A monomial consists of only one term.

Note: A term may consist of a group of letters and numerals not separated by a plus (+) or minus (-) sign.

Example: $\underbrace{2a^2}_{\text{one term}}$ monomial

The term may consist of numerals and letters. The combination of numbers and letters is considered a term. A polynomial/multinomial consists of two or more terms. Usually, there are two or three, but the number of terms for a polynomial/multinomial is infinite. Here are examples of polynomial and multinomial terms.

Examples: $\underbrace{3a^2 + 4b}_{\text{two terms}}$ binomial

$\underbrace{3a^2 + 4b - 3c^2}_{\text{three terms}}$ trinomial

A binomial is a polynomial/multinomial consisting of two terms separated by an operation sign, in this case plus or minus. A trinomial is a polynomial consisting of three terms separated by an operation sign, in this case a plus or minus sign. To see if you understand the definitions of the various types of numerical terms, try this challenge.

(Multiple Choice)

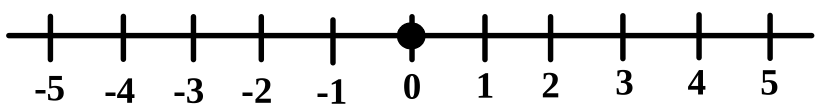
How many terms are there in a monomial?

a. One
b. Two
c. Three
d. Four

You are correct if you selected "a." A monomial consists of one term. Remember, a polynomial/multinomial consists of two or more terms separated by an operation sign. A trinomial is a polynomial consisting of three terms separated by an operation sign. If you answered incorrectly, review the section on numerical terms in this paragraph before continuing.

Now you must understand the use of reference as it applies to algebraic functions. A reference is a mark or sign, as a number, letter, or a symbol that directs you to a given point. See the example.

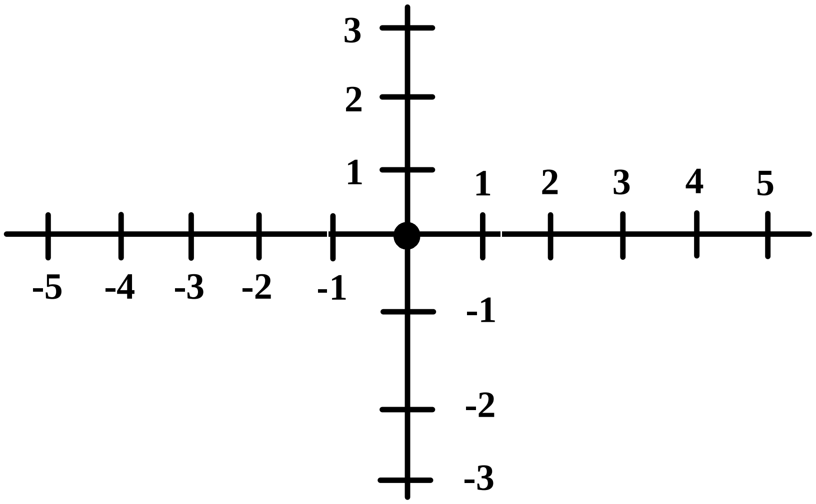
Example:



Real Number Line

The number line consists of a reference point (0) and equal value line segments on each side of zero (0). In this case, on the straight line, zero (0) is the reference point. The "0" is the center of the positive and negative vectors. A vector is a directed line segment showing magnitude and direction. There are 10 vectors other than the reference point "0" in the example. There are five negative vectors to the left of the reference point "0" and five positive vectors to the right of "0." Each vector represents a unit. Using a Cartesian Plane, notice the direction of positive and negative numbers in relation to their positions. See the example.

Example:



Cartesian Plane

The examples are used to show directions of positive and negative numbers. The concept of direction of positive and negative flow has many applications in electronics. Notice the positions of positive and negative numbers representing the vectors on the Cartesian Plane. You have just gone over material that explained line diagrams; those diagrams contained examples of the straight line and a Cartesian Plane. The purpose of the next challenge is to ensure that you understand the information. Try it!

(Fill in the blank)

A reference is a mark or sign, used as a number, letter, or a symbol that directs you to a ________.

a. diagram
b. symbol
c. negative point
d. specific point

If you selected "d," your answer is correct. You may continue. If it was not, review paragraph 1101 before continuing. Try the next challenge.

(Define)

Define the term vector.

____

____

____

You are correct if your response is the following: A vector is a directed line segment showing magnitude and direction. If you were correct, you may continue. If your response is incorrect, repeat paragraph 1101 before continuing.

1102. Solve Mathematical Problems

The next item of discussion explains how you must perform all operations. The conventional priority operation method will be used in this lesson and throughout the study unit. This method is selected to ensure that everyone working the same problem comes up with the same solution. The sequence of the conventional priority method is as follows:

1. Solve all roots and powers.
2. Perform all multiplication/division.
3. Perform all addition and subtraction.

Example:

$16 - 45 \div 3^2 * \sqrt{25} + 121 \div 11 + 1$ (Solve Powers)

$16 - 45 \div 9 * \sqrt{25} + 121 \div 11 + 1$

(Solve $16 - 45 \div 9 * 5 + 121 \div 11 + 1$

$16 - 5 * 5 + 11 + 1$

(Division)

$16 - 25 + 11 + 1$

$28 - 25$

(Multiplication)

3

By going through the example, you just solved an equation using the conventional method.

Try the following challenge to help you to memorize the sequence of conventional priority operation.

(Multiple Choice)

Choose the correct sequence you must use when performing the conventional priority operation method.

1. Perform all multiplication/division.
2. Reduce all roots and powers.
3. Perform all addition and subtraction.

a. 3, 2, 1

b. 1, 2, 3

c. 2, 1, 3

d. 3, 2, 1

If your answer is "c," you are correct. The first step is to solve all powers and roots, second, perform all multiplication/division; and third, perform all addition and subtraction. If your answer is incorrect, review paragraph 1102 before continuing. If your answer is correct, please continue.

Lesson Summary. In this lesson you learned the terms that are necessary to perform mathematical operations. You also learned the conventional priority operation method of solving problems. This approach will be used throughout the course. You should be able to apply it in real life situations as well as in practice. The lesson exercise will reinforce this skill. In the next lesson, you will study algebraic addition.

_ _

Lesson 1 Exercise: Complete items 1 through 9 by performing the actions required. Check your responses against those listed at the end of this lesson.

1. Which of the following sets of mathematical signs of operation have the same meaning in algebra as in arithmetic?

   a. $-, \bullet, 1, +, \pi$
   b. $\times, *, -, \pi, +$
   c. $*, +, \pi$
   d. $=, +, -, *, /$

2. Define the term "numerical expression."

   ____

   ____

   ____

3. Define the term "literal expression."

   ____

   ____

   ____

4. Which of the following signs must always precede the term of an equation?

   a. (+) b. (-)

5. Which of the following expressions is an example of like/similar terms?

a. $2a^2$
b. $2a$ $4a^4$
c. $2abc$ $2a^2bc$
d. $3a^4bc$ $5a^4bc$

6. How many terms are there in a monomial?

a. Four
b. Three
c. Two
d. One

7. A reference is a mark or sign, as a number, letter, or a symbol that directs you to a __________.

a. diagram
b. symbol
c. specific figure
d. specific point

8. Define the term vector.

____

____

____

9. Place the following steps in the correct sequence of performance using the conventional priority operations method.

1. Perform all multiplication/division.
2. Solve all roots and powers.
3. Perform all addition and subtraction.

a. 2, 1, 3
b. 1, 2, 3
c. 3, 2, 1
d. 3, 2, 1

Check your answers against those on the next page. Review those items that you missed before continuing on to the next lesson in this study unit.

Lesson 1 Exercise Solutions

	Answer	Reference
1.	d.	1101
2	Your answer should be or similar to the following: A numerical expression consists entirely of numbers separated by signs.	1101
3.	Your answer should be or similar to the following: A literal expressions consists of general numbers and letters.	1101
4.	b.	1101
5.	d.	1101
6.	d.	1101
7.	d.	1101
8.	Your answer should be or similar to the following: A vector shows magnitude and direction.	1101
9.	a.	1102

Lesson 2. ALGEBRAIC ADDITION AND SUBTRACTION

Introduction. In lesson 1 you learned the terms that will help you perform the requirements of this lesson and those that will follow. You must reference those terms and apply them as needed. In this lesson, you will add algebraic expressions including whole numbers, fractions, and algebraic expressions with exponents.

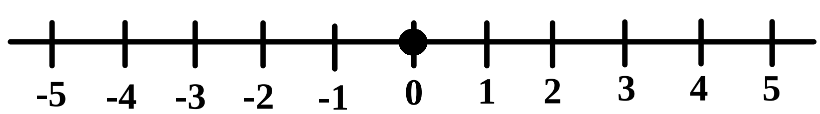
LEARNING OBJECTIVES

1. Add algebraic expressions without exponents, without the aid of reference.
2. Add algebraic expressions with exponents, without the aid of reference.
3. Subtract algebraic expressions without exponents, without the aid of reference.
4. Subtract algebraic expressions with exponents, without the aid of reference.
5. Check addition and subtraction expressions.

1201. Add Basic Expressions

Let us begin our discussion of adding basic expressions by defining the term absolute value. In lesson 1 you learned about real numbers using the straight line and Cartesian Plane diagrams. Now you can use those diagrams to help you understand the meaning of absolute value. Using the straight line diagram, the absolute value of zero (0) is zero.

Example:



The distance between each number to "0" is the magnitude of the number. The magnitude of a number is its absolute value.

Examples: The magnitude of

0 = 0
1 = 1
2 = 2
3 = 3

Try the following challenge.

(Define)

What is the absolute value of a number?

___

___

___

If you said, the magnitude of a number is its absolute value, you may continue. If your answer is incorrect, review the material that you just read before you continue.

We will refer to the term absolute value throughout this lesson and in future lessons now that you know its meaning. To add basic expressions, you must know the rules. There are three rules that you must follow when adding algebraic expressions. They are

- Rule 1 - To add two or more numbers with like signs, you must use the common sign. Refer to examples (a) and (b).

Examples: (a) (b)

$$\begin{array}{r} 2 \\ \underline{+2} \\ 4 \end{array} \qquad\qquad \begin{array}{r} -2 \\ (+)\ \underline{-2} \\ -4 \end{array}$$

- Rule 2 - Example (b) indicates that the sum of two negative numbers is the opposite of their absolute value.

- Rule 3 - The absolute value of the sum of two numbers, of which one is positive and one is negative, is the difference of their absolute values, and

IF	THEN
the positive number has the greater absolute value	the sum of the given number is positive.
the negative number has the greater absolute value	the sum of the number is negative.

Here are more examples of adding numbers with like and unlike signs. Study them.

Examples:

<u>Like</u>, using positive and negative signs

2+2 = 4
2.2+2 = 4.2
12+2 = 14
-2+(-2) = -4

<u>Unlike</u>, using unlike signs

(a)	(b)
7+(-3) = 4	5 Minuend
-9+3 = -6	<u>-3</u> Subtrahend
-7.6+2.2 = -5.4	2

<u>Note</u>: As in example (b), in using <u>unlike</u> signs, you simply change the sign and subtract.

Try the following challenges to see if you know your rules.

(Fill in the blank)

To add two or more numbers with like signs you use the ___________.

a. common sign
b. uncommon sign
c. opposite sign
d. smaller numeral sign

The correct answer is "a," common sign. If your answer is the same, you are correct; continue. If not, review the second part of paragraph 1201 before continuing. Try the next challenge.

(Solve these problems)		
(a)	(b)	(c)
36 +<u>27</u>	36 (+)<u>-18</u>	3*ab*+4*cd* (+)<u> *ab*-3*cd*</u>
(d)	(e)	(f)
4 (+)<u>-2</u>	-5 (-)<u>-7</u>	-2*ab*-3*cd* (-)<u>4*ab*+*cd* </u>

You are correct if your answer to each problem matches those listed below; continue. If all of your answers do not match those listed below, repeat paragraph 1201 and try the challenge again before continuing.

(Solve these problems)		
(a)	(b)	(c)
36 +<u>27</u> 63	36 (+)<u>-18</u> 18	3*ab*+4*cd* (+)<u> *ab*-3*cd*</u> 4*ab*+*cd*
(d)	(e)	(f)
4 (+)<u>-2</u> 2	-5 (-)<u>-7</u> 2	-2*ab*-3*cd* (-)<u>4*ab*+*cd* </u> -6*ab*-4*cd*

1202. Add Algebraic Expressions Using Exponents

Algebraic expression using exponents will allow you to more easily perform mathematical operations when using large numbers. This will become even more obvious when you multiply using exponents, but for now let us concentrate on addition. Look at the following chart to give you an idea of expressing and knowing the various parts of the algebraic expression. See figure 1-1.

EXPRESSION	READ	EXPONENT	BASE	MEANING
x^2	x squared	2	x	$x \bullet x$
y^3	y cubed	3	y	$y \bullet y \bullet y$
10^5	10 to the fifth power	5	10	$10 \bullet 10 \bullet 10 \bullet 10 \bullet 10$
3^6	3 to the sixth power	6	3	$3 \bullet 3 \bullet 3 \bullet 3 \bullet 3 \bullet 3$
z^1	z to the first power	1	z	

Fig 1-1. Examples of algebraic expressions using exponents.

Note: When an exponent is not indicated, it is understood to be 1.

Examples:

$$x = x^1$$
$$6 = 6^1$$

Now let us check your understanding of exponents. Try this challenge.

(Multiple Choice)

Select the base expression where the exponent is understood to be 1.

a. a^2b^3
b. $3cd^2$
c. x^3
d. 4^2

You are correct if your answer is "b." When there is no exponent indicated, it is understood that the exponent is 1. If you answered incorrectly, review paragraph 1202. If your answer is correct, you may continue.

The rules for performing algebraic addition **on terms containing exponents are**

- Rule 1 - The bases and the exponents must be the same (like terms).

  Example:

  a^2 (the base of the exponent is "a")

- Rule 2 - To add or subtract polynomials, arrange the terms in ascending or descending order. Perform algebraic addition where possible.

Note: This may require the insertion of a place holder. A place holder is merely a number where the value for addition and subtraction equals 0.

Example: Add

$2a^2 + 3ab + 4b^2c$ to $3a^2 + 3b^2c + 3b$

Set up as:

$$
\begin{array}{r}
2a^2 + 3ab + 4b^2c \\
(+)\underline{3a^2 + 0ab + 3b^2c + 3b} \\
5a^2 + 3ab + 7b^2c + 3b
\end{array}
$$

Sum of Absolute Values ^ $5a^2 + 3ab + 7b^2c + 3b$

(Fill in the blank)

The _________ and the _________ must be the same (like terms) before adding algebraic expressions with exponents.

a. bases, exponents
b. signs, terms
c. values, absolutes
d. terms, exponents

Try this challenge.

You are correct if your answer is "a." If it was not, review the rules for performing addition using algebraic expression with exponents.

Try these challenges.

(Multiple Choice)

To add or subtract polynomials, you must arrange the terms in which of the following ways?

a. Adjacent and descending
b. Ascending or descending
c. Squarely
d. Adjacent

The correct answer is "b." If your answer is different, review the section of paragraph 1202 that addresses adding and subtracting polynomials. Continue if your answer is correct.

(Multiple Choice)

When adding polynomials sometimes a place holder is needed. Choose the numeral or alphabetical expression that is used to represent a place holder.

a. *xy*
b. *ab*
c. 10
d. 0

The correct answer is "d." Zero is used as a place holder when adding polynomials. If your answer is correct, you may continue. If your answer is different, review the section in paragraph 1202 addressing space holders when adding polynomials.

Now shift your attention to working with very large or small equations. When performing a mathematical operation where very large or small numbers are used, it is useful to write the number as a power of 10.

Examples:

(a)	(b)
$3000 = 3 \times 10^3$	$.003 = 3 * 10^{-3}$

Follow this rule to add or subtract terms:

w Rule - To add or subtract terms using powers of 10, the bases and power of 10 must be the same.

Example: Add

$$\begin{array}{rcr} 2.34 \times 10^3 & = & 2.34 \times 10^3 \\ \underline{5433.89 \times 10^2} & = & \underline{54.3389 \times 10^3} \end{array}$$

It is usually possible to check the sum of two polynomials by assigning non-zero values to the variables.

Examples: (Using 2 as variable of x)

$$\begin{array}{rcr} -8x^2 + x + 5 & \rightarrow & -8(4) + (2) + 5 = -25 \\ \underline{4x^2 + 3x + 2} & \rightarrow & 4(4) + 3(2) + 2 = 24 \\ -4x^2 + 4x + 7 & \rightarrow & -4(4) + 4(2) + 7 = -1 \end{array}$$

Note: When x = 2, we know this does not ensure us that the sum is correct for all values of variables, but it does give reason for a degree of certainty of the correctness.

When performing an operation, it is important for you to know and remember that the letter in a rule is called a variable. A variable represents any number from a given set of numbers that are called a replacement set.

Try this challenge.

(Add and check the following polynomials)

$$\begin{array}{r} 3x^2 - 2x + 4 \\ \underline{2x^2 - 4x + 2} \end{array} \qquad \begin{array}{r} -5x^2 + 3x - 4 \\ \underline{-x^2 - 2x + 2} \end{array}$$

If your answers match the following, you are correct. If your answers differ, read paragraph 1302 beginning with the rule for adding and subtracting polynomials. If your answers are correct, please continue.

(Solution)

$$\begin{array}{r} 3x^2 - 2x + 4 \\ \underline{2x^2 - 4x + 2} \\ 5x^2 - 6x + 6 \end{array} \qquad \begin{array}{r} -5x^2 + 3x - 4 \\ \underline{-x^2 - 2x + 2} \\ -6x^2 + x - 2 \end{array}$$

When expressing the powers of 10, symbols are used. You may see these symbols in various equations. Study them so that you will recognize and know their meaning when you see them again.

10^{-12} = Pico ($\mu\mu$)
10^{-9} = nano (η)
10^{-6} = micro (μ)
10^{-3} = mili (m)
10^{3} = Kilo (k)
10^{6} = Mega (M)
10^{9} = Giga (G)
10^{12} = Tera (T)

To subtract polynomials, you must do the opposite of what you did when adding them. Adding and subtracting are inverse operations.

Example:

$12 - 4$ is the same as $12 + (-4)$

Study the following examples to ensure that you fully understand how to subtract polynomials.

Example:

Polynomial	Opposite
$4x^2$	$-(4x^2) or -4x^2$
$x^4 + 2y$	$-(x^2 + 2y), or -x^4 - 2y$
$-5x^3 + 6x - 2$	$-(-5x^3 + 6x - 2), or\ 5x^3 - 6x + 2$
$4xy - 2xy + 2$	$-(4xy - 2xy + 2), or -4xy + 2xy - 2$
$-8x + 6xy$	$-(-8x + 6xy), or\ 8x - 6xy$

To subtract polynomials, set them up in the same manner as in addition. Subtracting one polynomial from another is the same as adding the opposite of that polynomial.

Example: (a) (b)

$$\begin{array}{rr} & 7xy^3 \\ (-) & \underline{2xy^3} \\ & 5xy^3 \end{array} \qquad \begin{array}{rr} & 12x - 7xy^2 \\ (-) & \underline{5x - \ xy^2} \\ & 7x - 6xy^2 \end{array}$$

To check the accuracy of a subtraction problem, assign non-zero values to the variables as you did in addition.

Example: Values of variables x = 2, y = 3

$$\begin{array}{rl} & 12x - 5y \rightarrow 12(2) - 5(3) = 9 \\ (-) & \underline{4x - \ y \rightarrow \ 4(2) - \ (3) = 5} \\ & 8x - 4y \rightarrow \ 8(2) - 4(3) = 4 \end{array}$$

Try the following challenge.

(Fill in the blank)

Translate the following powers of 10 using the correct symbol(s) for each

10^{-12} = __________

10^{-9} = __________

10^{-6} = __________

10^{-3} = __________

10^{3} = __________

10^{6} = __________

10^{9} = __________

10^{12} = __________

If your response matches the one below you are correct; continue. If not, review the section addressing powers of 10 in paragraph 1202 before continuing.

10^{-12} = Pico ($\mu\mu$)
10^{-9} = nano (η)
10^{-6} = micro (μ)
10^{-3} = milli (m)
10^{3} = Kilo (k)
10^{6} = Mega (M)
10^{9} = Giga (G)
10^{12} = Tera (T)

(Subtract these polynomials)

(a)

$$\begin{array}{r} 2x+8 \\ x+2 \end{array}$$

(b)

$$\begin{array}{r} 10ab+4a-4c^2 \\ \underline{5ab+2a-\ c^2} \end{array}$$

(c)

$$\begin{array}{r} 2ab-12b+6c \\ \underline{ab-\ 6b+\ c} \end{array}$$

(d)

$$\begin{array}{r} 12x+6xy^2-3y \\ \underline{4x+2xy^2-\ y} \end{array}$$

If your answers to the problems match those below, you are correct. If not, review the section addressing subtracting polynomials in paragraph 1202.

(Solutions)

(a)

$$\begin{array}{r} 2x+8 \\ \underline{x+2} \\ x+6 \end{array}$$

(b)

$$\begin{array}{r} 10ab+4a-4c^2 \\ \underline{5ab+2a-\ c^2} \\ 5ab+2a-3c^2 \end{array}$$

(c)

$$\begin{array}{r} 2ab-12b+6c \\ \underline{ab-\ 6b+\ c} \\ ab-6b+5c \end{array}$$

(d)

$$\begin{array}{r} 12x+6xy^2-3y \\ \underline{4x+2xy^2-\ y} \\ 8x+4xy^2-2y \end{array}$$

Lesson Summary. In this lesson, you learned to add and subtract algebraic expressions with and without exponents. You learned to check the results of additions and subtractions to ensure their correctness. You learned the rules that will allow you to perform problem solving requirements. You will be able to apply the skills and knowledge as required. Lesson 3 will address multiplying and dividing algebraic expressions.

_ _

Lesson 2 Exercise: Complete items 1 through 9 by performing the action required. Check your responses against those listed at the end of this lesson.

1. The magnitude of a number is its __________ __________.

   a. partial value
   b. absolute value
   c. partial time
   d. absolute part

2. To add two or more numbers with like signs you must use the _______ _______.

   a. smaller number sign
   b. opposite sign
   c. uncommon sign
   d. common sign

3. Solve the addition problems.

(a)	(b)	(c)
$\begin{array}{r} 78 \\ \underline{54} \end{array}$	$\begin{array}{r} 66 \\ (+)\ \underline{94} \end{array}$	$\begin{array}{r} -9ab + 16cd \\ \underline{12ab - 6cd} \end{array}$

(d)	(e)	(f)
$\begin{array}{r} -8 \\ -\underline{13} \end{array}$	$\begin{array}{r} 9 \\ -\ \underline{6} \end{array}$	$\begin{array}{r} -13ad - 12cd \\ \underline{2ad - 7cd} \end{array}$

4. Select the base expression where the exponent is understood to be 1.

a. x^3
b. 4^2
c. $3cd^2$
d. a^2b^3

5. To add or subtract polynomials, you must arrange the terms in which of the following ways?

a. Ascending or descending
b. Adjacent and descending
c. Squarely
d. Adjacent

6. Choose the numeral or algebraic expression that is used to represent a place holder.

a. xy
b. ab
c. 10
d. 0

7. The ___________ and the ___________ must be the same (like terms) before adding algebraic expressions with exponents.

a. value, absolute
b. term, exponent
c. sign, term
d. base, exponent

8. Give the name and symbols of the following.

10^{12} = ___________
10^{9} = ___________
10^{6} = ___________
10^{3} = ___________
10^{-3} = ___________
10^{-6} = ___________
10^{-9} = ___________
10^{-12} = ___________

9. Subtract these polynomials

(a)

$$\begin{array}{r} 21ab + 6a - 11c^2 \\ \underline{3ab + 8a - \quad c^2} \end{array}$$

(b)

$$\begin{array}{r} 4x + 29 \\ \underline{x + 18} \end{array}$$

(c)

$$\begin{array}{r} 16x + 28xy^2 - 71y \\ \underline{8x + 12xy^2 + \ 90y} \end{array}$$

(d)

$$\begin{array}{r} 2ab - 14b + 4c \\ \underline{ab + \ 8b + \ 6c} \end{array}$$

Check your answers with those on the next page. Review those items that you missed before continuing on to the next lesson in this study unit

Lesson 2 Exercise Solutions

			References
1.	b.		1201
2.	d.		1201
3.	(a)	132	1201
	(b)	160	1201
	(c)	3*ab*+10*cd*	1201
	(d)	-21	1201
	(e)	3	1201
	(f)	-11*ad*-19*cd*	1201
4.	c.		1201
5.	a.		1201
6.	d.		1201
7.	d.		1201
8.	Tera (*T*)		1201
	Giga (*G*)		1201
	Mega (*M*)		1201
	Kilo (*K*)		1201
	milli (*m*)		1201
	micro (*u*)		1201
	nano (*n*)		1201
	Pico (*uu*)		1201
9.	(a)	$18ab - 2a - 10c^2$	1202
	(b)	$3x + 11$	1202
	(c)	$8x + 16xy^2 - 161y$	1202
	(d)	$ab - 22b - 2c$	1202

Lesson 3. ALGEBRAIC MULTIPLICATION AND DIVISION

Introduction. In lessons one and two you learned the basic foundations of algebraic mathematics. You must continue to build on your acquired foundation; lesson three will allow you to do so. You will be able to apply your acquired skills and knowledge in any given situation as required.

<u>LEARNING OBJECTIVES</u>

1. Calculate the product of positive and negative terms.

2. Calculate the quotient of positive and negative terms.

3. Reduce grouped quantities to their simplest form.

4. Apply the law of exponents when calculating the product of two or more quantities containing exponents.

5. Demonstrate that any number (except zero), raised to zero power has a value of one using the law of exponents.

6. Change the sign of an exponent (factoring).

7. Apply the law of exponents when calculating the power to which an original base must be raised, when given a power raised to a power.

8. Apply the law of exponents when calculating the quotient of two or more quantities containing exponents.

9. Demonstrate factoring exponents using a radical sign with the numerator as the power of the base and the denominator serving as the index of the radicand.

10. Demonstrate that any factor, under a radical, can be changed to a factor, with a fractional exponent, with the power base as the numerator and the index of the radicand as the denominator.

1301. Multiply and Divide Numbers with Signs

The purpose of multiplying numbers with signs is to obtain the product of two or more numbers. A product is defined as the result of multiplying two or more numbers. (We will discuss division and its result in the next paragraph.) There are three rules that you must remember when multiplying algebraic expressions.

These rules are

w Rule - The product of two numbers with like signs is positive.

Example:

(a)	(b)
3	-3
(*x*)3	(*x*)-3
9	9

Note: Remember, positive (+) number ∗ positive (+) number = positive (+) number.
Negative (-) number ∗ negative (-) number = positive (+) number

w Rule - The product of two numbers with unlike signs is negative.

Example:

(a)	(b)
3	−3
(*x*) − 3	(*x*)3
−9	−9

Note: Remember, positive (+) number ∗ negative (-) number = negative (-) number.

w Rule - The product of an even number of negative factors is positive, while the product of an odd number of factors is negative.

Example:

$$(-3) * (-3) * (-3) * (-3) = 81$$

$$3 * (-3) * (-3) * (-3) = -81$$

Try the following challenge to reinforce your memory and application of the rules.

(Multiply the following)

(a)	(b)	(c)
+16	-12	-51
(x) 3	(x)24	(x) 3

(d) (-12) x (-4) x (-2) =

(e) (10) x (-2) x (-2) =

If your answers to the problems are the same as those below, you are correct and may continue. If you answered any problem incorrectly, review paragraph 1301 before you continue.

(a)	(b)	(c)
+16	-12	-51
(x) 3	(x) 24	(x) 3
48	-288	-153

(d) (-12) x (-4) x (-2) = -96

(e) (10) x (-2) x (-2) = 40

It is easy to understand division operation once you have mastered multiplication of numbers with signs. To gain an understanding of calculating the quotient of positive and negative numbers, you must learn and keep in mind the rules that you must apply. The rules for dividing sign numbers are

- Rule - The quotient of two numbers having like signs is positive.

Example:

(a) $\frac{9}{3} = 3$

(b) $\frac{-9}{-3} = 3$

w Rule - The quotient of two numbers having unlike signs is negative.

Example:

(a) $\frac{-9}{3} = -3$

(b) $\frac{9}{-3} = -3$

Note: It is important to remember that dividing by zero is an undefined term.

Try these easy challenges.

(Fill in the blank)

When dividing, the quotient of two numbers with like signs is _________.

a. positive
b. negative
c. same
d. different

You are correct if your answer is "a." If not, read again the portion of paragraph 1301 that addresses calculating the quotient of positive and negative numbers.

(Fill in the blank)

When dividing, the quotient of two numbers with unlike signs is __________.

a. same
b. different
c. negative
d. positive

You are correct if your answer is "c." If you answered incorrectly, review the paragraph 1301 before continuing. If you are correct, continue.

1302. Group Quantities

Since you have been introduced to basic algebraic multiplication, you are now ready to reduce mathematical expressions contained in groups to their simplest form. First, you must know the signs and containers of groups. The following signs/symbols are containers of groups.

The signs of grouping are

a. Parentheses ( )
b. Brackets [ ]
c. Braces { }
d. Vinculum $\overline{\qquad\qquad}$
e. Radical $\sqrt{\qquad\qquad}$

To group, you must use sets of groups of numbers or combinations of letters and numbers in one of the containers. To reduce mathematical quantities contained in signs of grouping, you must perform the following steps in sequence.

1. Combine like terms inside the same signs of grouping when possible.

2. Remove signs of grouping one set at a time.

3. Parentheses or other signs of grouping preceded by a positive sign can be removed without changes.

4. Parentheses or other signs of grouping preceded by a negative (-) sign change the sign of every term within that sign or grouping.

5. A vinculum is a bar drawn over two or more algebraic terms to show that they are to be considered a single term.

6. If signs of grouping occur one with another, remove the inner group first and work your way out.

Examples:

(a)

6-{12+4[-3+8-(11-15) (10-6)]}
6-{12+4[-3+8-(-4) (4)]}
6-{12+4[-3+8-(-16)]}
6-{12+4[- 3+8+16]}
6-{12+4[5+16]}
6-{12+4*21}
6-{12+84}
6-{96}
6-96
-90

(b)

$$-6x-\left\{12xy+2\left[8x+4-\overline{6xy+12x}\right]\right\}$$
$$-6x-\left\{12xy+2\left[8x+4-6xy-12x\right]\right\}$$
$$-6x-\left\{12xy+2\left[-4x+4-6xy\right]\right\}$$
$$-6x-\left\{12xy-8x+8-12xy\right\}$$
$$-6x-\left\{-8x+8\right\}$$
$$-6x+8x-8$$
$$2x-8$$

Try the following challenge.

(Remove the signs of grouping)

(a)

$$2x+4\left[2-4\left(3y+5-\overline{3x+2y}\right)\right]$$

(b)

$$6a^2-4\left[-2+a^2-4+\left(5+\overline{7b+10-5}\right)\right]$$

If your answers match the following, you are correct; continue. If not, reread paragraph 1302. Try the challenge again before continuing. This will strengthen your knowledge for future operations.

(a)

$$2x + 4\left[2 - 4\left(3y + 5 - \overline{3x + 2y}\right)\right]$$
$$2x + 4[2 - 4(3y + 5 - 3x - 2y)]$$
$$2x + 4[2 - 4(y + 5 - 3x)]$$
$$2x + 4[2 - 4y - 20 + 12x]$$
$$2x + 4[-18 - 4y + 12x]$$
$$2x - 72 - 16y + 48x$$
$$50x - 16y - 72$$

(b)

$$6a^2 - 4\left[-2 + a^2 - 4 + \left(5 + \overline{7b + 10 - 5}\right)\right]$$
$$6a^2 - 4[-2 + a^2 - 4 + (5 + \overline{7b + 5})]$$
$$6a^2 - 4[-2 + a^2 - 4 + (5 + 7b + 5)]$$
$$6a^2 - 4[-2 + a^2 - 4 + (10 + 7b)]$$
$$6a^2 - 4[a^2 + 7b + 4 + 10 + 7b]$$
$$6a^2 - 4[a^2 + 7b + 4]$$
$$6a^2 - 4a^2 - 28b - 16$$
$$2a^2 - 28b - 16$$

1303. Calculate Exponents

In lesson 2 you learned to add expressions with exponents. Remember that you were told you would learn more about calculating exponents as we discussed the multiplication of them. In lesson 2 you also learned the basic rules relative to exponents. Those basic rules will be applied as we multiply base numbers with exponents. There are additional rules that you must learn and apply when appropriate. Study the following rules so that you can apply them when required.

w Rule - To find the product of two or more powers with <u>like</u> bases, add the exponents.

Examples:

(a)

$$a^m * a^n = a^{m+n}$$

(b)

$$3^2 * 3^2 = 9 * 9 = 81 \text{ or } 3^2 * 3^2 = 3^4 = 81$$

w Rule - To find the product of two or more quantities containing powers with <u>unlike</u> bases, multiply the bases to the common power.

Examples:

(a)

$$a^3 * b^3 = (ab)^3$$

(b)

$$2^2 * 3^2 = 4 * 9 = 36 \text{ or } 2^2 * 3^2 = (2 * 3)^2 = 6^2 = 36$$

(c)

$$2^2 * 3^2 * 2^2 = 4 * 9 * 4 = 144$$

or

$$2^2 * 3^2 * 2^2 = (2 * 3 * 2)^2 = 12^2 = 144$$

w Rule - To find the product of two or more like quantities containing powers of 10, treat the powers of 10 as exponents.

Example:

$$\begin{array}{r} 2.5 * 10^3 \\ \times 3.10 * 10^5 \\ \hline 7.75 * 10^8 \end{array}$$

w Rule - To raise a power of an exponent, keep the base and multiply the powers.

Example:

$$(a^m)^n = a^{m*n} = a^{mn}$$

Try the challenge to help you remember and apply the rules you have learned.

(Find the products of the following)

(a) $a^2 * a^3$

(b) $6^3 * 2 + 3 * 4 + 15$

(c)
$$\begin{array}{r} 7.5 * 10^2 \\ (*)\ .5 * 10^3 \\ \hline \end{array}$$

(d) $(2^3)^2$

If your answers match those listed below, you may continue. If not, review paragraph 1303 before continuing.

(a)

$a^2 * a^3 = a^5$

(b)

$6^3 * 2 + 3 * 4 + 15$

$216 * 2 + 3 * 4 + 15$

$432 + 12 + 15 = 459$

(c)

$$\begin{array}{r} 7.5 * 10^2 \\ (*)\ .5 * 10^3 \\ \hline 3.75 * 10^5 \end{array}$$

(d)

$(2^3)^2 = (8)^2 = 64$

or

$(2^3)^2 = (2)^{3*2} = 2^6 = 64$

Let us continue to learn our rules.

w Rule - Any factor of a fraction (numerical or literal) may be transferred from the numerator to denominator, or vice versa, by changing the sign of the exponent.

Examples:

(a)

$\frac{1}{3^2} = \frac{1}{9} \approx .111...$

$\frac{1}{3^2} = 3^{-2} \approx .111...$

(b)

$\frac{1}{a^n} = a^{-n}$

$a^{-2} = \frac{1}{a^2}$

Try this challenge.

Transfer the following fractions.

(a)

$\frac{2}{3} =$

(b)

$\frac{m^{-3}}{m} =$

(c)

$\frac{6y}{y} =$

If your answers match the following, you may continue. If not, review the material in paragraph 1303 that address the rule for transferring factors.

(a) (b) (c)

$\frac{2}{3} = 0.666...$ $\frac{m^{-3}}{m} = m^{-4}$ $\frac{6y}{y} = 6$

Lets look at the next few rules that pertain in general to exponent law.

- Rule - All numbers are understood to have an exponent of 1.

Example:

$2 = 2^1$

- Rule - Any number divided by itself equals 1.

Example:

$\frac{6}{6} = 1$

- Rule - Any number raised to the zero power = 1.

Example:

(a) $\frac{6^1}{6^1} = 6^{1-1} = 6^0 = 1$

(b) $\frac{a}{a} = \frac{a^1}{a^1} = a^{1-1} = a^0 = 1$ $(a \neq 0)$

Note: This rule does not apply to the number zero to the zero power since division by zero is considered undefined.

To ensure your understanding of the general rules, translate the equations in the following challenge.

(Translate according to exponent law)

(a) $\frac{12}{12} =$

(b) $4 =$

(c) $\frac{xy}{xy} =$

If your answers match those that follow, you are correct and you may continue. If not, review the section of paragraph 1303 that discusses general rules of exponent law before continuing to the next paragraph.

(a) (b)

(c) $\frac{12}{12} = \frac{12^1}{12^1} = 12^{1-1} = 12^0 = 1$ $\quad 4 = 4^1 \quad$ $\frac{xy}{xy} = \frac{xy^1}{xy^1} = xy^{1-1} = xy^0 = 1$

1304. Solve Radical Expressions and Radicands by Applying the Law of Exponents

By now you should have a basic understanding of calculating exponents. Now you will work with radicals. Remember, when you are required to square a number, it means raising it to the second power. See the example.

Example:

$5^2 = 25$

What you actually did was to multiply that number by itself. This is squaring the number. The opposite of squaring a number is finding the square root of a number. To find the square root, you must apply the inverse operation of squaring the number. The symbol that is used for square root is the radical sign. It is shown in figure 1-2.

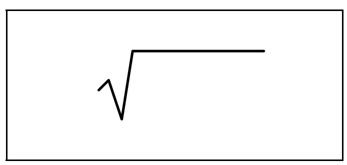


Fig 1-2. Radical sign.

Note: The square root of a number is a number that, when multiplied by itself, gives the original number.

The sign indicates the operation required. When a number or letter is added, it is called a radical.

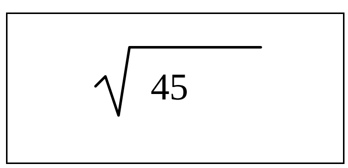


Fig 1-3. Radical.

Note: The required action is to find the square root of the number inside of the symbol.

The radical indicates the positive square root of the number. In figure 1-3 this is shown as $\sqrt{45}$.
It is normally called the principal square root. Sometimes that number may be called the radicand. In our lesson, we will call it the radicand. Try the next challenges.

(Fill in the blank)

The $\sqrt{\ }$ is called a __________.

a. radical sign
b. stem
c. square root
d. root

The correct response is "a." If your answer is correct try the next challenge. If not, read paragraph 1304 again before continuing.

(Multiple choice)

Select the symbol that symbolizes a radical.

a. $\sqrt{\ }$

b. $\sqrt{\ }$ +

c. $\sqrt{+}$

d. $\sqrt{81}$

The correct answer is "d." continue if your answer is correct. If you answered incorrectly, review paragraph 1304 before continuing.

(Fill in the blank)

The number inside of the sign is called a __________.

a. radical expression
b. radicand
c. radical
d. expression

The correct response is "b." The radicand is the principal square root. It is called the radicand. If your answer is correct, continue. If not, review paragraph 1304 before continuing.

Every positive number has two square roots, a negative square root and a positive square root.

Example:

The square root of 25 is 5
The square of 5 is 25
(5) (5) = 25
(-5) (-5) = 25

To indicate the negative square root, you must place the negative sign in front of the radical.

Example:

$-\sqrt{\ }$

Sometimes there is more than one number in a square root. When this occurs, always use the positive or principle root.

Example:

Positive $\sqrt{25}$ means $+\sqrt{25}$

Negative $(-)\sqrt{25}$ means -5 anc

$\frac{\text{Positive}}{\text{Negative}} \pm \sqrt{25}$ means ± 5

Note: A negative number has no real square root.

Square roots can be both rational and irrational. The following are examples of rational square roots.

Examples:

(a) $-\sqrt{25} = -5$

(b) $\sqrt{\frac{100}{9}} = \frac{10}{3}$

$\sqrt{\frac{100}{9}} = \frac{\sqrt{100}}{\sqrt{9}} = \frac{10}{3}$

(c) $\sqrt{10^2} = 10$

$\sqrt{10^2} = \sqrt{100} = 10$

There are also perfect square integers. A perfect square integer is an integer with a square root that is an integer.

Examples:

(a) $\sqrt{9} = 3$

(b) $\sqrt{144} = 12$

The square roots of other positive numbers are irrational.

Examples:

(a) $\sqrt{5}$

(b) $\sqrt{7}$

(c) $-\sqrt{13}$

(d) $-\sqrt{27}$

Note: Leave irrational square roots in radical form. You can *approximate* the value of the square root by finding the two consecutive integers that meet the following conditions. The square of the smaller integer will be less than the number whose square root you're looking for. The square of the larger integer will be greater than the number you want to find the square root of. The square root will be between the integers.

First you may want to guess the number. Next, divide the number by your guess. Finally, average the quotient and divisor.

Another area you need to understand about radical expressions is how they relate to exponents.

- Rule - To remove a radical sign from radicand or base number, you make a fractional exponent. Make a fractional exponent by using the exponent as the numerator of the fractional exponent and the index number as the denominator of the fractional exponent.

Examples:

(a) $\sqrt[n]{a^m} = a^{\frac{m}{n}}$

(b) $256^{\frac{1}{4}} = \sqrt[4]{256^1} = 4$

Try the following challenge to reinforce what you have learned in paragraph 1304.

Find the roots of the following:

(a)	(b)	(c)	(d)
$\sqrt{81}$	$-\sqrt{36}$	$\sqrt{2}$	$\sqrt[3]{\frac{27}{64}}$
(e)	**(f)**	**(g)**	**(h)**
$\sqrt[3]{27}$	$\pm\sqrt{16}$	$125^{\frac{1}{3}}$	$\sqrt{144}$

The following responses are correct. If your answers do not match all of the below, review paragraph 1304 before continuing. If all of your responses match, continue.

(a)	(b)	(c)	(d)
$\sqrt{81} = 9$	$-\sqrt{36} = -6$	$\sqrt{2} \approx 1.41$	$\sqrt[3]{\frac{27}{64}} = \frac{\sqrt[3]{27}}{\sqrt[3]{64}} = \frac{3}{4} = .75$
(e)	**(f)**	**(g)**	**(h)**
$\sqrt[3]{27} = 3$	$\pm\sqrt{16} = \pm 4$	$125^{\frac{1}{3}} = \sqrt[3]{125^{1}} = 5$	$\sqrt{144} = 12$

<u>Lesson Summary</u>. You have completed lesson 3. In this lesson you learned the terms and rules needed to perform electronic mathematical procedures. You learned to apply rules to calculate products and quotients of positive and negative terms. You learned to reduce grouped quantities to their simplest form. You learned to apply the law of exponents when calculating the product of two or more quantities containing exponents. You used the law of exponents to demonstrate the value of zero as an exponent. You learned to factor numbers and to apply the law of exponents when calculating the power to an original base when a given power is raised to a power. You learned to calculate the quotient of two or more quantities containing exponents. You learned to use radical signs with fractions and whole numbers. In the next lesson, you'll learn how to divide and multiply polynomials by monomials. The lesson exercise will serve as a measuring device to determine how well you understand what you learned. It will also point out those items that you need to review before you take the study unit exercise. Concentrate on each item carefully.

Lesson 3 Exercise: Complete items 1 through 12 by performing the action required. Check your responses against those listed at the end of this lesson.

1. The sign of the quotient of two numbers with like signs is __________.

   a. negative　　b. positive
   c. different　　d. same

2. The sign of the quotient of two numbers with unlike signs is __________.

   a. positive　　b. different
   c. same　　d. negative

3. Multiply the following:

(a)	(b)	(c)
-83	+19	-11
6	4	28

(d)

10 x (-7) x (-9) =

(e)

(-18) x (-3) x (-2) =

4. Remove the signs of groupings in the following equations.

(a)

$$6a^2 - 4[-2 + a^2 - 4 + (5 + \overline{7b + 10 - 5})]$$

(b) $2x + 4\left[2 - 4\left(3y + 5 - \overline{3x + 2y}\right)\right]$

5. Find the product of the following:

(a)

$a^4 * a^5 =$

(b)

$$\begin{array}{r} 9.1 * 10^4 \\ \underline{.5 * 10^2} \end{array}$$

(c)

$5^4 * 3 - 7 * 8 + 35$

6. Transfer the following fractions

(a)

$\frac{9y^2}{y^2}$

(b)

$\frac{5}{6}$

(c)

$\frac{n^{-8}}{n^2}$

7. Translate according to exponent law.

(a) $\frac{ab}{ab} =$

(b) $\frac{29}{29} =$

(c) 9=

8. To factor over integers you must rewrite the polynomial as the ____ only.

a. values of factors that have no coefficients
b. coefficients of factors with absolute value
c. sum of factors that have coefficients
d. product of factors that have integral coefficients

9. The $\sqrt{\ }$ is called a ___________.

a. stem
b. root
c. radical sign
d. square root

10. Select the symbol that symbolizes a radical.

a. $\sqrt{95}$

b. $\sqrt{+}$

c. $\sqrt{\ }$ +

d. $\sqrt{\ }$

11. The number inside of the sign is called a ___________.

a. radical
b. radical expression
c. expression
d. radicand

12. Find the indicated roots of the following.

(a) $\sqrt[3]{64}$

(b) $\pm\sqrt{64}$

(c) $216^{\frac{1}{3}}$

(d) $\sqrt{196}$

(e) $\sqrt{169}$

(f) $-\sqrt{49}$

(g) $\sqrt{3}$

(h) $\sqrt[3]{\frac{343}{512}}$

Check your answers with those on the next page. Review those items you missed before continuing to the next lesson in this study unit.

Lesson 3 Exercise Solution

			References
1.	b.		1301
2.	d.		1301
3.			
	(a)	-498	1301
	(b)	76	1301
	(c)	-308	1301
	(d)	630	1301
	(e)	-108	1301
4.			
	(a)	$2a^2 - 28b - 16$	1302
	(b)	$50x - 16y - 72$	1302
5.			
	(a)	a^9	1303
	(b)	$4.55 * 10^6$	1303
	(c)	1854	1303
6.			
	(a)	9	1303
	(b)	0.8333...	1303
	(c)	n^{-10}	1303
7.			
	(a)	$ab^{1-1} = 1$	1303
	(b)	$29^{1-1} = 1$	1303
	(c)	9^1	1303
8.	(d)		1304
9.	(c)		1304
10.	(a)		1304
11.	(d)		1304
12.			
	(a)	4	1304
	(b)	± 8	1304
	(c)	6	1304
	(d)	14	1304
	(e)	13	1304
	(f)	-7	1304
	(g)	≈ 1.732	1304
	(h)	$\frac{7}{8}$	1304

Lesson 4. MULTIPLICATION AND DIVISION OF MONOMIALS AND POLYNOMIALS

Introduction. In the previous lessons of this study unit, you learned the basic foundations of algebraic mathematics. You must continue to build on your acquired foundation. Lesson 4 will allow you to do so. You will be able to apply your acquired skills and knowledge in any given situation as required. In this lesson, you will learn to multiply and divide monomials and polynomials.

LEARNING OBJECTIVES

1. Identify monomials and polynomials.

2. Identify properties of real numbers.

3. Multiply monomials and polynomials.

4. Divide monomials and polynomials.

1401. Identifying Monomials and Polynomials

Before beginning to work with monomials and polynomials, you must understand the terms that are used during required operations. The following algebraic expressions are defined to help you identify, understand, and use them while performing the mathematical operations of multiplying and dividing monomials and polynomials.

a. Term. A term is a single expression containing literal and/or numerical parts not separated by a plus (+) or minus (-) sign.

Examples:

(a)	(b)	(c)
IR, or $I * R$	$-12a^3x^4y$	IE, or $I * E$

b. Monomials. A monomial is an expression consisting of one term not separated by a plus (+) or minus (-) sign.

Note: A single term is referred to as a monomial.

Examples:

(a)	(b)	(c)
I^2R	$4a^2x^2y$	$2\pi fl$

c. Polynomials. Polynomials are algebraic expressions consisting of two or more terms separated by a plus (+) and/or minus (-) sign.

Note: Polynomials are sometimes referred to as multinomials. This means the same thing.

Examples:

(a)	(b)	(c)
$2a+3b$	$3x^2-4y+2z^2$	$4a^2+12ab+9b^2-12$

d. Binomials. A binomial is a polynomial containing only two terms separated by a plus (+) or minus (-) sign.

Examples:

(a)	(b)	(c)
$2a+3b$	$3x^2-4y$	$4a^2+12ab$

e. Trinomial. A trinomial is a polynomial containing only three terms separated by a plus (+) and/or minus (-) sign.

Examples:

(a)	(b)	(c)
$2a+3b-c$	$3x^2-4y+2z^2$	$4a^2+12ab+9b^2$

Since you should understand the different types of monomial and polynomial expressions, you should be ready to try the following challenge to reinforce your memory.

(Fill in the blank)

Identify the term that best describes the algebraic expression.

(a) $4a^2 + 12ab + 9b^2$ ______________

(b) $3x^2 - 4y$ ______________

(c) $4a^2 + 12ab + 9b^2 - 12$ ______________

(d) $2\pi fl$ ______________

If your answer to the challenge is the same as follows, you are correct and may continue. If your answer differs, review paragraph 1401 before continuing.

Remember these terms and their meanings. You will use them as you perform the various operations (add, subtract, multiply, or divide).

(a) $4a^2 + 12ab + 9b^2$	Trinomial
(b) $3x^2 - 4y$	Binomial
(c) $4a^2 + 12ab + 9b^2 - 12$	Polynomial
(d) $2\pi fl$	Monomial

f. <u>Degree of Monomials</u>. The degree of a monomial is determined by the highest power number of any literal factor.

Examples:

$5x^2$ is degree two $\quad x^2 = x * x =$ two literals

$9ab^3$ is degree three $\quad b^3 = b * b * b =$ three literal

$6a^5b^6c^4$ is degree six, not degree 3 or 15.

<u>Note</u>: The degree is determined by the highest power of the literal factors, not by the number of literal factors.

g. Degree of Polynomials. The degree of a polynomial is determined by finding the term with the most literal factors.

Examples:

$2a^2 + 3ab + b^2$ is degree two

$5xy^2 + 2x^2y + y^4$ is degree four

h. Quadratic Polynomial. A quadratic polynomial is a polynomial of the second degree (degree two).

1402. Multiplication Properties of Real Numbers

Now that you know the basic algebraic expressions relating to monomials and polynomials you are ready to learn more specific terms that will enable you to perform various operations using them. You will also be introduced to properties for real and literal numbers. Study the following properties so that you can apply them when required.

a. Associative property. The regrouping of factors does not affect the product for all real numbers.

Example:

$$x(yz) = (xy)z$$

b. Commutative property. The order of the factors does not affect the product for real numbers.

Example:

$$xy = yx$$

c. Distributive property. If multiplication is performed on grouped quantities containing plus (+) and/or minus (-) signs, the multiplication must be distributed to all quantities in the sign of grouping.

Example:

$$x (y + z) = xy + xz$$

Try the following challenge on properties of real numbers.

(Fill in the blank)

Identify the term that best describes the expressions.

(a) $7(a+t)=7a+7t$ ____________ ____________

(b) $3(xy)=(3x)y$ ____________ ____________

(c) $IR=RI$ ____________ ____________

If your answers to the challenge are the same as the following, you are correct and may continue. If your answers are incorrect, review paragraph 1402 before continuing.

(a)	$7(a+t)=7a+7t$	Distributive property
(b)	$3(xy)=(3x)y$	Associative property
(c)	$IR=RI$	Commutative property

1403. Identify Monomial and Polynomial Products

Now you are ready to use the three major properties to find the product of algebraic expressions. You will start by multiplying monomials. To obtain monomial products, follow four easy steps. First, determine the sign, then multiply the numerical coefficients, next multiply the literal coefficients, and finally, multiply these two products together. Follow this example so that you can understand each step as it is performed.

Examples:

$3x * (-5xy)$:

Determine sign	=	$(+) * (-) = (-$
		$3 * -5 = -15$
Numerical values	=	$x * x * y = x^2y$
Literal values	=	$-15x^2y$
Product	=	

Using the four steps in the example, find the product of each equation in the following challenge.

Find the products of the following monomial equations.

(a)	(b)	(c)
$(-7IR)(-12E^2) =$	$(-4a^2b)(3bc^2) =$	$\left(3x^2y^2\right)\left(6xy^2\right)$

If your answer to the challenge is the same as follows, you may continue. If you answered any equation in the challenge incorrectly, review paragraph 1403 before continuing.

(a)

$$(-7IR)(-12E^2) =$$
$$(-) * (-) = (+)$$
$$7 * 12 = 84$$
$$I * R * E^2 = E^2IR$$
$$= 84E^2IR$$

(b)

$$(-4a^2b)(3bc^2) =$$
$$(-) * (+) = (-)$$
$$4 * 3 = 12$$
$$a^2 * b * b * c^2 = a^2b^2c^2$$
$$= -12a^2b^2c^2$$

(c)

$$\left(3x^2y^2\right)\left(6xy^2\right) =$$
$$(+) * (+) = (+)$$
$$3 * 6 = 18$$
$$x^2 * x * y^2 * y^2 = x^3y^4$$
$$= 18x^3y^4$$

You are now ready to multiply a polynomial by a monomial. Multiply each term of the polynomial by the monomial using the distributive property. If you need to review the distributive property, it was covered in paragraph 1402. As you can see in the following examples, writing the product of each operation will help you maintain the appropriate signs.

Examples:

(a)

$$4x(2m - 3n)$$
$$= 4x(2m) - 4x(3n)$$
$$= 8mx - 12nx$$

(b)

$$-9c(8c - 9cL + 3L)$$

$$= -9c(8c) - (-9c)(9cL) + (-9c)(3L)$$

$$= -72c^2 - (-81c^2L) + (-27cL)$$

$$= -72c^2 + 81c^2L - 27cL$$

Try the following challenge on finding the products of monomials and polynomials.

Find the products of the following equations.

(a) $2x(3x - 2y) =$

(b) $-b(a^2 + ab + b^2) =$

(c) $2y(4x^2y + 7xy + 3y^2) =$

If your answers to the challenge are the same as follows, you are correct and may continue. If you answered any part of the challenge incorrectly, review paragraph 1403 before continuing.

(a)

$$2x(3x - 2y$$
$$= 2x(3x) - 2x(2y)$$
$$= 6x^2 - 4xy$$

(b)

$$-b(a^2 + ab + b^2)$$
$$= -b(a^2) - b(ab) - b(b^2)$$
$$= -a^2b - ab^2 - b^3$$

(c)

$$2y(4x^2y + 7xy + 3y^2$$
$$= 2y(4x^2y) + 2y(7xy) + 2y(3$$
$$= 8x^2y^2 + 14xy^2 + 6y^3$$

1404. Find Binomial Products

You can find a product for two binomials by using a procedure called the FOIL method. The abbreviation FOIL provides a four step procedure to multiply binomials more efficiently. You multiply in this order, the first terms, the outer terms, the inner terms, the last terms, and then add the resultant polynomial. In the example below, you can see the FOIL method step by step.

Example:

$$(a+b)(x+y$$

F first terms $(a+b)(x+$

$= ax$

O outer terms $(a+b)(x+$

$= ax + ay$

I inner terms $(a+b)(x$

$= ax + ay + bx$

L last terms $(a+\underline{b})(x+$

$= ax + ay + bx + \underline{by}$

Note: If possible, you need to combine any like terms by algebraic addition.

Find the products of two binomials in the following challenge.

Find the products of the following binomial equations.

(a) $(x-y)(3x-2y) =$

(b) $(a+b)^2 =$

(c) $(2x+y)(x-2y) =$

If your answers to the problems are the same as those below, you are correct and may continue. If you answered any problem incorrectly, repeat the section on multiplying two binomials in paragraph 1404 before continuing.

(a)

$$(x-y)(3x-2y$$
$$3x^2-2xy-3xy+2y^2$$
$$3x^2-5xy+2y^2$$

(b)

$$(a+b)^2$$
$$(a+b)(a+b)$$
$$a^2+ab+ab+b^2$$
$$a^2+2ab+b^2$$

(c)

$$(2x+y)(x-2y$$
$$2x^2-4xy+xy-2y^2$$
$$2x^2-3xy-2y^2$$

1405. Polynomial Products

The expressions you will most often find yourself dealing with are polynomials times polynomials. Although these expressions can look intimidating at times, they are really no more difficult than any other monomial/polynomial combination you have done. An aid in finding the product of two polynomials is to arrange each polynomial in the same order, in either ascending or descending degrees of one literal term. Once you have arranged the polynomials, multiply each term of the multiplicand by each term of the multiplier. Combine like terms if possible and you have your product of two polynomials. Study the following example so you can find the product of two polynomials.

Example:

Multiply

$$(3a^2-2ab+b^2)(3ab-2b^2+a^2)$$

$$\begin{array}{r} 3a^2-2ab+b^2 \\ a^2+3ab-2b^2 \\ 3a^4-2a^3b+a^2b^2 \qquad\qquad\qquad \\ +9a^3b-6a^2b^2+3ab^3 \qquad \\ -6a^2b^2+4ab^3-2b^4 \\ \hline 3a^4+7a^3b-11a^2b^2+7ab^3-2b^4 \end{array}$$

Try the following challenge to reinforce all that you have learned in this paragraph 1405.

Find the products of the following polynomial equations.

(a)

$$(2x+3+5y)(2x+3-5y)=$$

(b)

$$(5x+2y+4)(5x+2y-3)=$$

If your answers to the equations are the same as those below you are correct. You may continue. If you answered either equation incorrectly, rework them and check for any mathematical mistakes and repeat paragraph 1405 before continuing.

(a)

$$\begin{array}{l} 2x+3+5y \\ \underline{2x+3-5y} \\ 4x^2+6x+10xy \\ \quad 6x+0xy+9+15y \\ \quad\quad -10xy+0-15y-25y^2 \\ 4x^2+12x \quad +9 \quad -25y^2 \end{array}$$

$$\text{Answer} = 4x^2+12x+9-25y^2$$

(b)

$$\begin{array}{l} 5x+2y+4 \\ \underline{5x+2y-3} \\ 25x^2+10xy+20x \\ \quad 10xy+0x+4y^2+8y \\ \quad\quad \underline{-15x+0y^2-6y-12} \\ 25x^2+20xy+5x+4y^2+2y-12 \end{array}$$

$$\text{Answer} = 25x^2+20xy+5x+4y^2+2y-12$$

1406. Divide a Monomial by a Monomial

Since you have completed paragraphs 1402 through 1405 and have an understanding of multiplying and finding the products of monomials and polynomials, you should now focus on learning to divide them. You will begin the learning process by dividing monomials by monomials. This process is very similar to traditional arithmetic division. To divide monomials by monomials you must divide the numerical coefficients and affix the correct sign. Then you must divide the literal factors and combine the signs of the numerical coefficient and literal terms to reach the solution. The process may seem a little much for you to grasp for now. Read the paragraph again. Then follow each step of the example below. Make sure you grasp each step before moving to the next.

Example:

Divide

$$\frac{12x^2y^2}{-3xy}$$

Numerical values = $\frac{12}{-3} = -4$

Literal values = $\frac{x^2y^2}{xy} = xy$

Product = $-4xy$

Now that you have studied the example work the challenge.

Find the result by dividing the following equations.

(a)

$$\frac{27a^3b^2}{-3ab^2} =$$

(b)

$$\frac{-81x^4}{9x^2} =$$

If your answers to the equations in the challenge are the same as those that follow, you are correct and may continue. If you answered either equation incorrectly, review paragraph 1406 before continuing.

(a)

$$\frac{27a^3b^2}{-3ab^2} = -9a^2$$

(b)

$$\frac{-81x^4}{9x^2} = -9x^2$$

1407. Divide Polynomials by Monomials

You are now ready to divide polynomials by monomials. The process is the same as in paragraph 1406, dividing monomials by monomials. You must divide each term of the polynomial (the numerator) by the monomial (the denominator). Then, you should write each result in succession using the correct sign and perform simple addition. Study the example. You may need to read the steps again as you study the example.

Example:

$$(45a^4b - 27a^3b^2 - 9a^2b^3 + 6ab^4 - 8) \div (-3ab^2)$$

<u>Note</u>: Divide each term of the polynomial by the monomial.

$$\frac{45a^4b}{-3ab^2} - \frac{27a^3b^2}{-3ab^2} - \frac{9a^2b^3}{-3ab^2} + \frac{6ab^4}{-3ab^2} - \frac{8}{-3ab^2}$$

$$(-15a^3b^{-1}) - (-9a^2) - (-3ab) + (-2b^2) - \left(-\frac{8}{3ab^2}\right)$$

$$-15a^3b^{-1} + 9a^2 + 3ab - 2b^2 + \frac{8}{3ab^2}$$

Now that you have studied the example, work the following challenge provided for you.

Divide the following expressions.

(a)

$$-27x^3y^2z^5 + 3x^4y^2z^4 - 9x^4y^3z^5 \div -3x^3y^2z^4$$

(b)

$$8a^2b^3c - 12a^3b^2c^2 + 4a^2b^2c \div 4abc^2$$

If your answers to expressions in the challenge are the same as those that follow, you are correct and may continue. If you answered either part incorrectly, go back, check your work, and review paragraph 1407 before continuing.

(a)

$$-27x^3y^2z^5 + 3x^4y^2z^4 - 9x^4y^3z^5 \div -3x^3y^2z^4$$

$$\left(\frac{-27x^3y^2z^5}{-3x^3y^2z^4}\right) + \left(\frac{3x^4y^2z^4}{-3x^3y^2z^4}\right) + \left(\frac{-9x^4y^3z^5}{-3x^3y^2z^4}\right) =$$

$$(9z) + (-x) + (3xyz$$

$$9z - x + 3xyz$$

(b)

$$8a^2b^3c - 12a^3b^2c^2 + 4a^2b^2c \div 4abc^2$$

$$\left(\frac{8a^2b^3c}{4abc^2}\right) + \left(\frac{-12a^3b^2c^2}{4abc^2}\right) + \left(\frac{4a^2b^2c}{4abc^2}\right) :$$

$$(2ab^2c^{-1}) + (-3a^2b) + (abc^{-1})$$

$$2ab^2c^{-1} - 3a^2b + abc^{-1}$$

1408. Divide a Polynomial by a Polynomial

When you divide polynomials, arrange the divisor in descending (normal), or ascending order of degree of one of the literal factors if possible. Use zeros as space holders for any degree missing between the higher and lower degrees. Arrange the dividend the same as you did for the divisor. Read through each step carefully. If you have difficulty following the example, read the paragraph again.

Note: Remember your definitions for exponent degrees discussed in paragraph 1401.

Example:

$$(19a^3 + 51a^2 + 12a^5 - 18a^4 + 61) \div (-2a + a^2 + 3)$$

$$a^2 - 2a + 3 \overline{)\,12a^5 - 18a^4 + 19a^3 + 51a^2 + 0a + 61}$$

Note: The problem has been set up in long division, descending order, and a place holder of 0 has been placed in the dividend.

- Step 1 You need to divide the first term of the divisor into the first term of the dividend to find the first term of the quotient. Then multiply the entire divisor by the first term of the quotient.

$$\frac{12a^5 \text{ dividend}}{a^2 \text{ divisor}} = 12a^3$$

$$12a^3(a^2 - 2a + 3) = 12a^5 - 24a^4 + 36a^3$$

$$\begin{array}{r} 12a^3 \qquad\qquad\qquad\qquad\qquad\qquad \\ a^2 - 2a + 3 \overline{)\,12a^5 - 18a^4 + 19a^3 + 51a^2 + 0a + 61} \\ -(12a^5 - 24a^4 + 36a^3) \qquad\qquad\qquad\quad \end{array}$$

- Step 2 Just as you would do in long division, subtract this product from the dividend by changing the signs and adding algebraically.

$$\begin{array}{r} 12a^3 \qquad\qquad\qquad\qquad\qquad\qquad \\ a^2 - 2a + 3 \overline{)\,12a^5 - 18a^4 + 19a^3 + 51a^2 + 0a + 61} \\ \underline{-12a^5 + 24a^4 - 36a^3} \qquad\qquad\qquad\quad \\ 6a^4 - 17a^3 + 51a^2 + 0a + 61 \end{array}$$

w Step 3 You should repeat the first two steps until there is no remainder or a remainder of which the first term cannot be evenly divided by the first term of the divisor.

$$
\begin{array}{r l}
 & 12a^3 + 6a^2 \\
\cline{2-2}
a^2 - 2a + 3 & \overline{)\,12a^5 - 18a^4 + 19a^3 + 51a^2 + 0a + 61} \\
 & \underline{-12a^5 + 24a^4 - 36a^3} \\
 & \qquad 6a^4 - 17a^3 + 51a^2 + 0a + 61 \\
 & \qquad \underline{-6a^4 + 12a^3 - 18a^2} \\
 & \qquad\qquad -5a^3 + 33a^2 + 0a + 61
\end{array}
$$

$$
\begin{array}{r l}
 & 12a^3 + 6a^2 - 5a \\
\cline{2-2}
a^2 - 2a + 3 & \overline{)\,12a^5 - 18a^4 + 19a^3 + 51a^2 + 0a + 61} \\
 & \underline{-12a^5 + 24a^4 - 36a^3} \\
 & \qquad 6a^4 - 17a^3 + 51a^2 + 0a + 61 \\
 & \qquad \underline{-6a^4 + 12a^3 - 18a^2} \\
 & \qquad\qquad -5a^3 + 33a^2 + 0a + 61 \\
 & \qquad\qquad \underline{5a^3 - 10a^2 + 15a} \\
 & \qquad\qquad\qquad 23a^2 + 15a + 61
\end{array}
$$

$$
\begin{array}{r l}
 & 12a^3 + 6a^2 - 5a + 23 \\
\cline{2-2}
a^2 - 2a + 3 & \overline{)\,12a^5 - 18a^4 + 19a^3 + 51a^2 + 0a + 61} \\
 & \underline{-12a^5 + 24a^4 - 36a^3} \\
 & \qquad 6a^4 - 17a^3 + 51a^2 + 0a + 61 \\
 & \qquad \underline{-6a^4 + 12a^3 - 18a^2} \\
 & \qquad\qquad -5a^3 + 33a^2 + 0a + 61 \\
 & \qquad\qquad \underline{5a^3 - 10a^2 + 15a} \\
 & \qquad\qquad\qquad 23a^2 + 15a + 61 \\
 & \qquad\qquad\qquad -23a^2 + 46a - 69 \\
 & \qquad\qquad\qquad\qquad 61a - 8
\end{array}
$$

w Step 4 $61a$ does not divide evenly by a^2, so the quotient is the algebraic sum of each of the partial quotients obtained and (if there is a remainder) a fraction with the remainder in the numerator and the divisor in the denominator.

$$\frac{12a^5 - 18a^4 + 19a^3 + 51a^2 + 61}{a^2 - 2a + 3} = 12a^3 + 6a^2 - 5a + 23 + \frac{61a - 8}{a^2 - 2a + 3}$$

Now that you have studied the example, work this challenge.

Find the result by dividing the following equations.

(a)

$$\beta^2 - 2\beta \overline{\left)\, 6\beta^4 + 2\beta^3 - 6\beta - 16\right.}$$

(b)

$$x^2 - 2 \overline{\left)\, x^4 + 3x^2 + 5\right.}$$

If your answer to the challenge is the same as those below, you are correct and may continue. If you answered either part incorrectly, go back check your work over and review paragraph 1408 before continuing.

(a)

$$\begin{array}{r} 6\beta^2 + 14\beta + 28 \\ \beta^2 - 2\beta \overline{\left)\, 6\beta^4 + 2\beta^3 + 0\beta^2 - 6\beta - 16\right.} \\ -6\beta^4 + 12\beta^3 \\ 14\beta^3 + 0\beta^2 \\ -14\beta^3 + 28\beta^2 \\ 28\beta^2 - 6\beta \\ -28\beta^2 + 56\beta \\ 50\beta - 16 \end{array}$$

$$= 6\beta^2 + 14\beta + 28 + \frac{50\beta - 16}{\beta^2 - 2\beta}$$

(b)

$$\begin{array}{r} x^2 + 5 \\ x^2 - 2 \overline{\left)\, x^4 + 3x^2 + 5\right.} \\ \underline{-x^4 + 2x^2} \\ 5x^2 + 5 \\ \underline{-5x^2 + 10} \end{array}$$

$$= x^2 + 5 + \frac{15}{x^2 - 2}$$

Lesson Summary. In this lesson you have identified monomials and polynomials. You covered the properties of real numbers. You also covered how to multiply and divide monomials and polynomials. These basic concepts will be applied time after time when finding the parameters of electrical and electronic circuits.

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Lesson 4 Exercise: Complete items 1 through 12 by performing the action required. Check your responses against those listed at the end of this lesson.

1. From the following list of terms, select the one that best describes each expression: monomial, polynomial, binomial, trinomial

(a) $9x^2y^2 + 5xy + 15y^2 - 28$ ______________

(b) $2j^2 + 4c^4$ ______________

(c) $6x^2 13xy + 5y^2$ ______________

(d) $\pi r^2 h$ ______________

2. Identify the property that best describes the operation being performed to the algebraic expressions

(a) $AZ = ZA$ _________________________

(b) $12(ab) = (12a)b$ _________________________

(c) $k(j + l) = kj + kl$ _________________________

(Fill in the blank)

3. Find the product of the following monomial equation.

$$\left(9x^3y^4\right)\left(15x^5y^3\right) =$$

4. Find the product of the following monomial equation.

$$(-6I^2R)(-15E^2IR) =$$

5. Find the product of the following equation.

$$-b^2(a^3 + 3a^2b^2 + 4ab^2 + 2b^2) =$$

6. Find the product of the following equation.

$$6y^2\left(3x^2y+4x^2y^2+9xy+11y^2\right)=$$

7. Find the product of the following binomial equation.

$$\left(-2x+y\right)\left(x+2y\right)=$$

8. Find the product of the following binomial equation.

$$(a+b)^2=$$

9. Find the product of the following polynomial equation.

$$\left(15x + 4y + 2\right)\left(2x + 5y - 5\right) =$$

10. Find the result by dividing the following equation.

$$\frac{36a^4b^5c^6}{-1.5a^3b^5} =$$

11. Find the result by dividing the following equation.

$$16a^5b^7c^4 + 12a^4b^6c^3 + 8a^{11}b^5c \quad \div 4a^2b^2c^2$$

12. Find the result by dividing the following equation.

$$\beta^2 + 2\beta \overline{\smash{\big)}\, 6\beta^4 + 2\beta^3 - 6\beta + 16}$$

Check your answers with those on the next page. Review those items that you missed before continuing to the study unit exercise.

Lesson 4 Exercise Solutions

		References
1.		
	(a) Polynomial	1401
	(b) Binomial	1401
	(c) Trinomial	1401
	(d) Monomial	1401
2.		
	(a) Commutative property	1402
	(b) Associative property	1402
	(c) Distributive property	1402
3.	$135x^8y^7$	1403
4.	$90E^2I^3R^2$	1403
5.	$-a^3b-3a^2b^4-4ab^4-2b^4$	1403
6.	$18x^2y^3+24x^2y^4+54xy^3+66y^4$	1403
7.	$-2x^2-3xy-2y^2$	1404
8.	$a^2+2ab+b^2$	1404
9.	$30x^2+83xy+20y^2-10y-71x-10$	1405
10.	$-24ac^6$	1406
11.	$4a^3b^5c^2+3a^2b^4c+2a^9b^3c^{-1}$	1407
12.	$6\beta^2-10\beta+20+\frac{46\beta+16}{\beta^2+2\beta}$	1408

UNIT SUMMARY

This study unit provided you with instruction on the mathematical terms, signs, and order of operation. You also covered the basic principle of grouping, products, quotient of terms, and principles of polynomials. Study Unit 2 will introduce you to factoring and fractions. You will then see how these basic principles are applied. Before continuing to Study Unit 2, complete the unit exercise.

Study Unit 1 Exercise: Complete items 1 through 20 by performing the action required. Check your responses against those listed at the end of this study unit exercise.

1. Simplify. $5+4\left[3-6-\left(\sqrt{16}-5^2 * \overline{4-3}\right)\right]$

a. 67
b. 162
c. 79
d. 77

2. Reduce. $\sqrt{16a^3b^2}$

a. $8a^2b$
b. $4ab\sqrt{a}$
c. $4a^2\sqrt{ab^2}$
d. $8ab\sqrt{a}$

3. Multiply. $(a+2)(a-5)$

a. $a^2-3a-10$
b. $a^2-7a-10$
c. $a^2+7a+10$
d. $a^2+7a-10$

4. Divide. $(3a^3+4a^2+45)\div(a+3)$

a. $3a^2+5a-15$
b. $3a^2-5a+15$
c. a^2+a-15
d. a^2-15

5. Express using positive exponents. $\frac{a^{-3}}{b^2c^{-4}}$

a. $\frac{b^2}{a^3c^4}$
b. $\frac{c^4}{a^3b^2}$
c. $\frac{b^2c^4}{a^3}$
d. $\frac{b^2}{a^3c^4}$

6. Divide. $\frac{6a^2b^3c}{2ab^2}$

a. $3a^3b^5c$ c. $8a3b^5c$

b. $3ac$ d. $3abc$

7. Add.

$$\begin{array}{r} 2ab+3a^2b-ab^2+4 \\ \underline{(+)7ab^2-2ab+a^2b-5} \end{array}$$

a. $4a^2b+6ab^2-1$ c. $5ab+a^2b-1$

b. $2a^2b+4ab-8ab^2+9$ d. $9a^2b^3-a+ab-1$

8. Multiply. $12ac(5a+3c-4)$

a. $60a^2c-36a^2c^2+48ac$ c. $60a^2c+36ac^2-48ac$

b. $72a^2c-36ac^2-44ac$ d. $9a^2b^3-a+ab-1$

9. Divide. $25a^3b^2+5a^2b-20ab \div 5ab$

a. $5a^2+a+4$ c. $5a^2b-a+4$

b. $5ab+a+4$ d. $5a^2b+a-4$

10. Add.

$$\begin{array}{r} 14 * 10^3 \\ \underline{+.15 * 10^5} \end{array}$$

a. $2.9 * 10^4$ c. $1.415 * 10^3$

b. $1.415 * 10^8$ d. $2.9 * 10^3$

11. Multiply.

$$\begin{array}{r} 7 * 10^2 \\ \underline{2.2 * 10^{-3}} \end{array}$$

a. $1.54 * 10^{-6}$ c. 1.54

b. $1.54 * 10^{-5}$ d. $1.45 * 10^5$

12. Subtract. $\begin{array}{r} 12x^2 + 4x - 15 \\ \underline{(-)6x^2 + 8x - 10} \end{array}$

a. $6x^2 + 8x - 25$
c. $18x^2 + 12x - 25$

b. $6x^2 - 4x - 5$
d. $6x^2 - 4x + 5$

13. Raise the monomial to the power shown. $(3a^2b^3c)^3$

a. $9a^5b^6c^4$
c. $27a^5b^6c^4$

b. $27a^6b^9c^3$
d. $9a^6b^9c^3$

14. Multiply.

$$\left(x + 2y\right)\left(x^2 - 3xy + y\right)$$

a. $x^3 - x^2y + xy + 2y^2 - 6y^2x$
c. $x^3 - x - 4y^2$

b. $x^3 + 2x^2y - 4y^3x^3$
d. $x^3 - 2x^2y + 4y^2x$

15. Multiply. $(a + b)(a - b)$

a. $a^2 - ab + b^2$
c. $a^2 + b^2$

b. $a^2 + ab + b^2$
d. $a^2 - b^2$

16. Reduce. $\sqrt[3]{8a^6b^3}$

a. $a^2b\sqrt{8}$
c. $2a^2b$

b. $2a^2b\sqrt{b}$
d. $2ab^2$

17. Multiply.

$$-3a^2 * 5b^2 * -2ab$$

a. $30a^3b^3$
c. $-30a^3b^3$

b. $36a^2b^3$
d. $38a^3b^2$

18. Reduce.

$$5 + 6 * 3 + \sqrt[3]{27} \div 3 - 2$$

a. 22
b. 32
c. 24
d. 18

19. Subtract.

$$\begin{array}{r} 3a^2b^2 + 2ab - 20 \\ \underline{(-) - 2a^2b^2 - 3ab + 5} \end{array}$$

a. $a^2b^2 - ab - 15$
b. $5a^2b^2 + 5ab - 25$
c. $-a^2b^2 + ab + 15$
d. $-5a^2b^2 - 5ab + 25$

20. Subtract.

$$\begin{array}{r} 526.7 * 10^3 \\ \underline{(-)4.375 * 10^5} \end{array}$$

a. $530.2 * 10^3$
b. $531.075 * 10^3$
c. $8.92 * 10^2$
d. $8.92 * 10^4$

Check your answers with those on the next page. Review those items that you missed before continuing on to the next study unit.

Study Unit 1 Exercise Solutions

		Reference
1.	d.	1302
2.	b.	1304
3.	a.	1401
4.	b.	1407
5.	b.	1303
6.	d.	1406
7.	a.	1201
8.	c.	1401
9.	d.	1407
10.	a.	1202
11.	c.	1303
12.	b.	1202
13.	b.	1303
14.	a.	1405
15.	d.	1405
16.	c.	1403
17.	a.	1303
18.	a.	1304
19.	b.	1202
20.	d.	1303

STUDY UNIT 2

FACTORING AND FRACTIONS

Introduction. In Study Unit 1, you learned basic information that will enable you to perform basic mathematical operations. In this study unit, you will learn to apply those same operations using fractions. You will also be able to factor. This study unit covers mixed fractions, addition, subtraction, multiplication, and division. You must fully understand the material in this study unit before you will be able to begin studying equations in Study Unit 3.

Lesson 1. INTRODUCTION TO FACTORING

LEARNING OBJECTIVES

1. Express any given whole number as a product of its prime factors.

2. Express any given polynomial whose terms contain common monomial factors as a product of its prime factors.

3. Factor certain expressions that are the product of two binomials (the sum, the difference, or the sum and difference of two binomial terms).

4. Factor any given trinomials using the trial and error method.

5. Reduce any given fraction to its simplest form by canceling like factors in the numerator and denominator.

6. Given two or more fractions, convert them to equivalent fractions having a common denominator by finding the lowest common multiple of the original denominators.

7. Express any given mixed number an equivalent improper fraction.

8. Convert any given improper fraction to a mixed number by dividing the numerator by the denominator.

2101. Understanding Factoring

You will find that a good understanding of factoring will not only help explain numerical equations but will also serve as a useful tool in managing large, more complex equations. The basic information that you will need to perform such operations is discussed in this lesson. To factor a number simply means that you find a group of numbers which, multiplied together, give the number. The product will result in the original number that you begin with.

Examples:

(a) $\underbrace{6 \bullet 5}_{\text{factors of 30}} = \underbrace{30}_{\text{product of 6 and 5}}$

(b) $\underbrace{\text{Product of 3, 5, and 2}}_{30} = \underbrace{\text{3, 5, and 2}}_{\text{Factors of 30}}$

Now, let us define and explain a few terms you will use when factoring.

a. Integer. Simply a number of a set.

Example: -3, -2, -1, 0, 1, 2, 3, -a, -b, -c, a, b, or c

b. Integral factor. A number that divides evenly into another number, or integer.

Note: In the first example of this paragraph, the product 30 was used and its integral factors shown. The integer 4 is not an integral factor of 30, because 4 does not divide evenly into 30. To find an integral factor of a number, start by dividing by 1 and all number combinations that divide evenly.

Example:
Integral Factor

(a) $\underbrace{\text{1, 2, 3, 4, 6, and 12}}_{\text{Factors of 12}} \qquad \underbrace{12}_{\text{Products of 1, 2, 3, 4, 6, and 12}}$

Note: The numeral one (1) is an integral factor of every integer.

c. Prime number. A prime number is a positive (+) integer greater than 1 with only two different integer factors. This means that only the integer 1 and the integer itself are evenly divided into the prime number.

Example:

Set of prime numbers

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31...

d. Greatest common factor (GCF). The largest number in two sets that are common to two given numbers.

Example:

Sets	Given numbers
1, 2, 3, 4, **6**	24
1, 2 **6**	18

Try these challenges to make sure that you know the terms you will need when you factor.

(Fill in the blank)

A number of a set is called an __________.

The correct answer is an integer, if you answered correctly, you may continue. If not, read the definition for this term again. Then, you may continue to the next challenge.

(Fill in the blank)

A number that divides evenly into another number, or integer, is called a (an) __________.

The correct answer is integral factor. If you answered correctly, you may continue. If not, read the definition again before continuing.

(Multiple Choice)

Which of the following sets are examples of prime numbers?

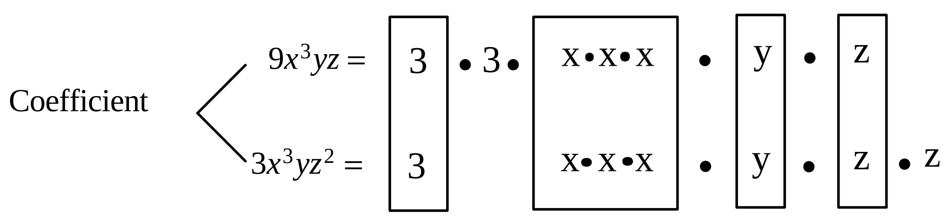
a. -2, -3, 8, 11
b. -1, 2, 3, 5
c. 2, 3, 5, 7
d. 1, 2, 3, 5

You are correct if your answer is "c." If not, read the definition of prime numbers again and study the example before continuing.

2102. Factor Monomials and Polynomials

Now you are ready to factor monomials. Realize that you simply multiply a number by another number to form a product. To see how to factor a polynomial, look at the following example. This process requires you to find the greatest common factor. Let's see how to do that.

Example:



$3x^3 \bullet y \bullet z = 3x^3yz$

GCF $= 3x^3yz$ — $3x^3yz$ is the largest factor common to both $9x^3yx$ and $3x^3yz^2$.

<u>Note</u>: Any factor of a product is the coefficient of the product of the remaining factors.

Study the following example using a negative integer.

Example:

$-4a^3b =$	2	•	2	•		$a \bullet a \bullet a$	$\bullet b$
$16a^3b =$	2	•	2	$\bullet 2 \bullet 2 \bullet$		$a \bullet a \bullet a$	$\bullet b$

GCF = 2^2a^3b

Try the following challenge to give you practice in factoring monomials.

(Show the greatest common factor GCF of the following monomials.)

(a)

$6xy$
$+12xy$

(b)

$10x$
$+3x$

(c)

$4xy$
$+5xy$

Your answer to the factors in the challenge should match those that follow. If your answers are correct, continue. If not, review paragraph 2101 before continuing.

(a)

$6xy = 2*3 \;*\; x*y$
$12xy = 2*3*2*x*y$

$2*3*x*y(1+2)$
$6xy(3) = 18xy$

GCF = $6xy$

(b)

$10x = 2*5*x$
$3x = 3 * x$

$x(2*5+3)$
$x(10+3) = 13x$

GCF = x

(c)

$4xy = 2*2*x*y$
$5xy = 5 * x*y$

$xy(2*2+5)$
$xy(4+5) = 9xy$

GCF = xy

With your newly acquired knowledge, you are ready to factor polynomials. When you transform a given polynomial into a product of other polynomials, the process is called polynomial factoring. You already have an understanding of factoring in relation to multiplying. However, study the example on the next page for reinforcement.

Example:

Multiplying	Factoring
$3(x+v) = 3x+3$	$3x+3v = 3(x+v$
$2(x-1) = 2x-2$	$2x+-1 = 2(x-1)$
$r(s+t) = rs+rt$	$rs+rt = r(s+t)$

To factor polynomials, you must remember the following three important things:

(1) The first example is factoring over integers. When you factor over integers, you must rewrite the polynomial as the product of factors that have only integral coefficients.

Examples:

(a)	(b)
$4ab+4b$	$a^3 - a$
$4ab+4b = 4(ab+b)$	$a^3 - a = a(a^2 - 1)$
Common monomial factor is 4.	Common monomial is a .

(2) The next example is a polynomial that has no coefficients except for 1 and itself. To factor, study the example.

Example:

$2a^3 + 4a^4 + 2a$
$2a^3 + 4a^4 + 2a = 2a(a^2 + 2a^3 + 1)$

Common monomial is $2a$.

(3) The final example shows two terms having no common factors.

Example:

$3a+2b$
$3a+2b = \underline{3a}\ \ \underline{2b}$ No common factors

<u>Note</u>: Therefore, $3a + 2b$ is a prime polynomial over the integers.

To reinforce what you have learned, try the following challenge.

(Fill in the blank)

To factor over integers you must rewrite the polynomial as the __________ of factors that have __________ __________ only.

a. values, no coefficients
b. coefficients, integral coefficients
c. product, integral coefficients
d. sum, integral coefficients

The correct response is "c." Continue if your answer is correct. If not, review the section of paragraph 2102 addressing polynomials before you continue.

It is always a good idea to check your factoring. You do so by multiplying the factors and comparing the product to the original polynomial.

Example:

$$12a^2 + 3a^4 - 2a^3 = a^2(12 + 3a^2 - 2a)$$

$$12a^2 + 3a^4 - 2a^3$$

$$(2 \bullet 2 \bullet 3 \bullet a \bullet a) + (3 \bullet a \bullet a \bullet a \bullet a) - (2 \bullet a \bullet a \bullet a)$$

$$GCF = a \bullet a = a^2$$

Multiplying the factors $a^2(12 + 3a^2 - 2a) = 12a^2 + 3a^4 - 2a^3$

The following challenge will allow you to practice what you have learned about factoring polynomials. Try it!

(Factor)

Factor the following polynomials. If the polynomial cannot be factored over an integer, write that it is a prime polynomial.

(a)	(b)	(c)
$x+3x=$	$2x-3x^2=$	$3ab+9a=$
(d)	(e)	(f)
$a+b-c^2=$	$4a^2b+6ab=$	$3x+9xy=$

Check your work to ensure your answers are correct. If they match those listed below, you may continue. If not, review paragraph 2102 before continuing.

(a)	(b)	(c)
$x+3x=x(1+3)$	$2x-3x^2=x(2-3x)$	$3ab+9a=3a(b+3)$
(d)	(e)	(f)
$a+b-c^2=$ Prime Polynomial	$4a^2b+6ab=2ab(2a+3)$	$3x+9xy=3x(1+3y)$

2103. Factoring Polynomial Squares

To factor special polynomials known as "sum of two squares," you first have to determine the signs. Once you have determined the signs, place a parenthesis around them. You are now ready to extract the square root of the first and last term. Now verify by multiplying and ensuring the product equals the expression you started with. Study the example on the next page.

Example: $(a^2 + 2ab + b^2)$

Determine signs $(\ +\ )(\ +\ )$

Extract the square root $(a + b)(a + b)$

Note: Use the FOIL method covered in Study Unit 1, lesson 4, paragraph 1404 to multiply two binomials and verify the product.

Now you are ready to factor the difference of two squares. This is done in the same manner as the sum of two squares. Once you have determined the signs, place parenthesis around them. You are now ready to extract the square root of the first and last terms. Verify by multiplying to ensure the product equals the expression you started with originally. Study the example.

Example: $(a^2 - 2ab + b^2)$

Determine signs $(\ -\ )(\ -\ )$

Extract the squares $(a - b)(a - b)$

Note: Use the FOIL method covered in Study Unit 1, paragraph 1404 to multiply two binomials and verify the product.

The last variation of polynomial squares is to factor the sum and difference of two squares. Determine the signs and place parentheses around them. Extract the square root of the first and last terms. Now verify by multiplying and ensuring the product equals the expression you started with originally. Study the example.

Example: $a^2 - b^2$

Determine signs $(\ -\ )(\ +\ )$

Extract the squares $(a - b)(a + b)$

Note: Use the FOIL method covered in Study Unit 1, lesson 4, paragraph 1404 to multiply two binomials and verify the product.

Try the following challenge to reinforce your memory on factoring polynomial squares.

(Factor)

Factor the following polynomial squares.

(a)	(b)	(c)
$x^2 - 2xy + y^2$	$9a^2 - 9b^2$	$4x^2 + 8xy + 4y^2$

Check your work to ensure your answers are correct. If they match those listed below, you may continue. If not, review paragraph 2103 before continuing.

(a)	(b)	(c)
$x^2 - 2xy + y^2$	$9a^2 - 9b^2$	$4x^2 + 8xy + 4y^2$
$(\ -\ )(\ -\ )$	$(\ -\ )(\ +\ )$	$(\ +\ )(\ +\ )$
$\left(x-y\right)\left(x-\right.$	$(3a-3b)(3a+3b)$	$\left(2x+2y\right)\left(2x+2\right.$

2104. Factor Trinomials

Now that you have finished with polynomial squares, you are ready to factor trinomials. In the preceding paragraph, the integers used were all square integral factors. This was done to make the problems easier to solve so you would concentrate on the different types of polynomial squares. You will find most problems have integers of varying values. You will apply all the rules you have learned up to this point using the trial and error method of factoring. Just as you did with the polynomial squares, the first step is to determine signs required to solve the trinomial. The next step in this operation is to determine all factors that will result in the first and last terms. The last step is to plug in all variations until you have the two binomials that will satisfy the original trinomial. Study the example on the next page. It will prepare you to solve the challenge that follows.

Example:

$$15x^2 + 24x - 12$$

Determine the sign (+) (-)

Determine all factors

15 = 1 * 15
= 3 * 5
= 5 * 3
= 15 * 1

12 = 1 * 12
= 2 * 6
= 3 * 4
= 4 * 3
= 6 * 2
= 12 * 1

Plug in all variations until solved.

$$(1x+12)(15x-1) \neq 15x^2 + 24x - 12$$

$$(3x+4)(5x-3) \neq 15x^2 + 24x - 12$$

Solution - $(3x+6)(5x-2) = 15x^2 + 24x - 12$

Try the following challenge.

(Factor)

Factor the following trinomials.

(a) $10x^2 + 7xy + y^2$

(b) $2x^2 - 2xy - 24y^2$

If your answers match the following, you are correct and may continue. If not, read paragraph 2104 again. Try the challenge again before continuing. This will strengthen your knowledge for future operations.

(a)	(b)
$10x^2+7xy+y^2$	$2x^2-2xy-24y^2$
$(\ +\ )(\ +\ )$	$(\ +\ )(\ -\ )$
$(5x+y)(2x+y$	$(2x+6y)(x-4y$
	$2(x+3y)(x-4y)$

2105. Reducing Fractions

Most of the formulas and equations in electronics will involve a quotient or fraction of some kind. Knowing how to reduce fractions to their simplest terms will help you. It also makes the equations more manageable. To reduce to the simplest terms you must cancel out like factors in both the numerator and denominator. After canceling out when and where required, you are ready to perform the required operation (add, subtract, multiply, or divide). In this case, you are now ready to multiply. Study the example that follows.

Example: $$\frac{36}{150}=\frac{2*2*3*3}{2*3*5*5}$$

Cancel like factors - $$\frac{\cancel{2}*2*\cancel{3}*3}{\cancel{2}*\cancel{3}*5*5}=\frac{2*3}{5*5}$$

Multiply - $$\frac{2*3}{5*5}=\frac{6}{25}$$

The process of simplifying literal fractions is the same as for numerical fractions. You factor both the numerator and denominator to prime numbers. You will use the distributive law of algebra to extract the highest number of factors from the numerator and the denominator. Once again, cancel like factors in the numerator and denominator. Then, multiply the remaining factors of the numerator, then multiply factors of denominator. Study the examples.

Examples:

(a)

$$\frac{a^2b}{ab^2}=\frac{\cancel{a}*a*\cancel{b}}{a*\cancel{b}*b}=\frac{a}{b}$$

$$\frac{6a^2b^3-9ab^2}{39ab-12a^3b^2}$$

$$= \frac{(2*3*a*a*b*b*b)-(3*3*a*b*b}{(3*13*a*b)-(2*2*3*a*a*a*b*b}$$

Factor out the highest common factor.

$$= \frac{(3*a*b*b)(2*a*b-3)}{(3*a*b)(13-2*2*a*a*b)}$$

Cancel like factors.

$$= \frac{(\cancel{3}*\cancel{a}*\cancel{b}*b)\ (2*a*b-3)}{(\cancel{3}*\cancel{a}*\cancel{b})\ (13-2*2*a*a*b)}$$

$$= \frac{b(2ab-3)}{(13-4a^2b)}$$

<u>Note</u>: You can cancel like terms in the numerator and denominator only if the terms are not separated by a plus (+) or a minus (-) sign.

Try the following challenge.

(Simplify)

Reduce the following equations.

(a)

$$\frac{55x^2y^3-35xy}{5xy} =$$

(b)

$$\frac{a-b}{a^2-b^2} =$$

If your answers match the following, you are correct and may continue. If not, read paragraph 2105 again. Try the challenge again before continuing. This will strengthen your knowledge for future operations.

(a)

$$\frac{55x^2y^3-35xy}{5xy} =$$

$$= \frac{(5*11*x*x*y*y*y)-(5*7*x*y}{5*x*y}$$

$$= \frac{(5*x*y)[(11*x*y*y)-(}{5*x*y}$$

$$= \frac{\cancel{(5*x*y)}\ (11*x*y*y) - (7)}{\cancel{5*x*y}}$$

$$= 11xy^2 - 7$$

(b)

$$\frac{a-b}{a^2-b^2}$$

$$= \frac{\cancel{(a-b)}}{\cancel{(a-b)}\,(a+b)} = \frac{1}{(a+b)}$$

$$\frac{a-b}{(a+b)(a-b)} = \frac{1}{a+b}$$

w Rule - Remember there are three signs to a fraction, sign of the fraction, sign of the numerator and sign of the denominator. You can change any two signs of the fraction without changing the value of the fraction. Study the example.

Example: $+\frac{+2}{+4} = +\frac{+1}{+2}$

$$-\frac{-2}{+4} = -\frac{-1}{+2} = +\frac{1}{2}$$

2106. Combining Fractions

Fractions can be combined (added and subtracted) ONLY if they have a common denominator. It is easiest if this denominator is the LOWEST COMMON DENOMINATOR (LCD) of the two fractions. The LCD is the smallest number into which the denominator of every fraction being combined is evenly divisible. This is the same thing as saying that the LCD is the LOWEST COMMON MULTIPLE (LCM) of the denominators. So, to combine fractions, you should first reduce each fraction to its lowest terms. This will help make the numbers you are working with as small as possible. Next, find the LCM of the denominators and convert each fraction to a new but equal fraction having this LCM as its denominator. Then, add or subtract the fractions by adding or subtracting their numerators. To find the LCM of denominators, begin by factoring the denominators into their prime factors.

The LCM will have every factor as many times as each denominator has that factor. No doubt this all seems very abstract so let's look at some examples!

Example: Find the LCM of and combine the fractions $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{9}$.

Denominators	Prime factors
3	3
4	2*2
9	3*3

Count the number of times each factor appears in any denominator.

Factor:	Times it appears
3	1
2 * 2	2
3 * 3	2

The LCM needs to have 3 twice and two twice or 3 times 3 times 2 times 2. This allows each denominator to divide into the LCM evenly with no remainder. The factors of the LCM are thus 2 * 2 * 3 * 3 and the LCM is 36. Now, you can convert each fraction to an equivalent fraction with a denominator of 36. Multiply both the numerator and denominator by the LCM.

$$\frac{1}{3} = \frac{?}{36}\left(\frac{36}{3} = 12\right)$$

Therefore:

$$\frac{1*12}{3*12} = \frac{12}{36}$$ or: $$\frac{1}{3} = \frac{x}{36}$$

The remaining fractions can be written as: $3x = 36 \quad x = \frac{36}{3} = 12$

$$\frac{1*9}{4*9} = \frac{9}{36}$$

$$\frac{1*4}{9*4} = \frac{4}{36}$$

If we wished to add the fractions, we would have $\frac{1}{3} + \frac{1}{4} + \frac{1}{9} = \frac{12+9+4}{36} = \frac{2}{3}$

You will use the same process for converting literal fractions as you do for numerical fractions. Look over the example on the following page.

Example: $\frac{a}{xy}, \frac{b}{x^2}, \frac{c}{xy^2}$

Denominators	Prime factors
xy	$x * y$
x^2	$x * x$
xy^2	$x * y * y$

$$LCM = x * x * y * y = x^2y^2$$

All fractions converted

$$\frac{a}{xy} = \frac{a*x*y}{x*x*y*y} = \frac{axy}{x^2y^2}$$

$$\frac{b}{x^2} = \frac{b*y*y}{x*x*y*y} = \frac{by^2}{x^2y^2}$$

$$\frac{c}{xy^2} = \frac{c*x}{x*x*y*y} = \frac{cx}{x^2y^2}$$

Try the following challenge to make equivalent fractions with the lowest common denominators. This challenge will reinforce your knowledge in this area.

(Solve)

Make equivalent fractions using the LCD.

(a) $\frac{3}{4}, \frac{4}{5}, \frac{3}{9} =$

(b) $\frac{18}{24}, \frac{7}{8}, \frac{6}{9} =$

Check your work to ensure your answers are correct. If they are correct, you may continue. If not, review paragraph 2106 before continuing.

(Solutions)

(a)

$\frac{3}{4}, \frac{4}{5}, \frac{3}{9} =$

$= \frac{3}{4}, \frac{4}{5}, \frac{1}{3}$

$\frac{45}{60}, \frac{48}{60}, \frac{20}{60}$

(b)

$\frac{18}{24}, \frac{7}{8}, \frac{6}{9} =$

$= \frac{3}{4}, \frac{7}{8}, \frac{2}{3}$

$\frac{18}{24}, \frac{21}{24}, \frac{16}{24}$

2107. Mixed Numbers and Improper Fractions

You will also find when dealing with electronic equations that you must convert mixed numbers to equivalent improper fractions. If you treat the mixed number as two fractions and convert to like fractions over a common denominator, all you have to do is add them. Look at the example provided showing a mixed number converted to an equivalent improper fraction.

Example: $5\frac{2}{3}$

$5\frac{2}{3} = \frac{5}{1} * \frac{3}{3} +$

$= \frac{15}{3} + \frac{2}{3}$

$= \frac{15+2}{3} = \frac{17}{3}$

$= \frac{17}{3}$

Try the following challenge converting mixed numbers to improper fractions. This challenge will reinforce your knowledge in this area.

(Solve)

Convert into improper fractions.

(a) $10\,\frac{11}{13} =$

(b) $4\,\frac{7}{8} =$

(c) $8\,\frac{5}{6} =$

Check your answers to make sure they are correct. If they match the responses below, you are correct, and may continue. If not, review paragraph 2107 before continuing.

(Solutions)

(a)

$10\,\frac{11}{13}$

$10\,\frac{11}{13} = \frac{10}{1} * \frac{13}{13} + \frac{11}{13}$

$= \frac{130}{13} + \frac{11}{13}$

$= \frac{130+11}{13} = \frac{143}{13}$

(b)

$4\,\frac{7}{8}$

$4\,\frac{7}{8} = \frac{4}{1} * \frac{8}{8} + \frac{7}{8}$

$= \frac{32+7}{8} = \frac{39}{8}$

$= \frac{32+7}{8} + \frac{7}{8}$

(c)

$8\,\frac{5}{6}$

$8\,\frac{5}{6} = \frac{8}{1} * \frac{6}{6} + \frac{5}{6}$

$= \frac{48}{6} + \frac{5}{6}$

$= \frac{48+5}{6} = \frac{53}{6}$

2108. Improper Fractions to Mixed Numbers

Now that you have mastered changing mixed numbers to improper fractions, we will cover the inverse operation. You will learn to perform long division using whole numbers. If you remember back in Study Unit 1, lesson 4, solving polynomial equations can be more difficult. The steps you must take will be to divide the numerator by the denominator. The even number of times the denominator goes into the numerator becomes the whole number.

The remainder becomes the fractional part of the mixed number. Look at the examples on how to convert mixed numbers.

Example:

(a)

$\frac{17}{3}$

$3\overline{)17}$ = 5 with a remainder of 2

$= 5\frac{2}{3}$

(b)

$\frac{a^2+1}{a-2}$

$$
\begin{array}{r}
a+2 \\
a-2\overline{)a^2+0a+1} \\
a^2-2a \\
2a+1 \\
2a-4 \\
5
\end{array}
$$

$= a+2+\frac{5}{a-2}$

Try out the following challenge converting improper fractions to mixed numbers. This challenge will reinforce your knowledge in this area.

(Solve)

Convert into mixed fractions.

(a) $\frac{39}{4}$

(b) $\frac{123}{5}$

(c) $\frac{2x^2-5}{x-2}$

Check your work to ensure your answers are correct and if they match the following responses. If they are correct, please continue. If not, review paragraph 2108 before continuing.

(Solutions)

(a)

$\frac{39}{4}$

$\frac{39}{4} = 9\frac{3}{4}$

(b)

$\frac{123}{5}$

$\frac{123}{5} = 24\frac{3}{5}$

(c)

$\frac{2x^2-5}{x-2}$

$$\begin{array}{r} 2x+4 \\ x-2 \overline{\big) 2x^2+0x-5} \\ \underline{2x^2-4x} \\ 4x-5 \\ \underline{4x-8} \\ 3 \end{array}$$

$= 2x + 4 + \frac{3}{x-2}$

Lesson Summary. You have completed lesson 1 of Study Unit 2. In this lesson you learned the processes of factoring whole numbers, monomials, binomial, trinomials, and polynomials. You learned to factor polynomial squares. You learned to factor expressions that are the product of two binomials (the sum, the difference, or the sum and difference of two binomial terms). You learned to reduce any given fraction to its simplest form/term. You learned, when given two or more fractions, to convert them to equivalent fractions having a common denominator by finding the lowest common multiple of the original denominator. You learned to convert mixed numbers to equivalent improper fractions and back by manipulating the denominators and numerators. In the next lesson, you will learn how to add and subtract fractions. Before you continue on to the next lesson, you need to complete the lesson exercise. The lesson exercise will serve as a measuring device and it will also point out those items that you need to review before you take the study unit exercise. Concentrate on each item carefully.

Lesson 1 Exercise: Complete items 1 through 11 by performing the action required. Check your responses against those listed at the end of this lesson.

1. Integers can be both __________ or negative numbers.

2. Two numbers of a set are called an __________.

3. A number that divided evenly into another number, or integer is called __________.

4. Find GCF for the following monomials.

(a)	(b)
$6x^2y^2$	$4xy$
$24x^2y^2$	$2axy$

5. Factor the following polynomials. If the polynomial cannot be factored over an integer, state that the polynomial is prime.

(a)	(b)	(c)
$9a^3b^2 + 6a^2b^2 + 2b^4 =$	$x^2 + xy + y =$	$6x^2 + 7x^3 =$

6. Factor the following polynomials squares.

(a) $81x^2 + 162xy + 81y^2$

(b) $x^2 + 2xy + y^2$

(c) $25a^2 - 25b^2$

7. Factor the following trinomials.

(a) $3x^2 + 8xy + 4y^2$

(b) $10x^2 + 19xy + 6y^2$

8. Reduce the following equations.

(a) $\frac{a^2+2ab+b^2}{a^2-b^2}$

(b) $\frac{55x^2y^3z+75xy}{5xy} =$

9. Make equivalent fractions with LCD.

(a)

$\frac{8}{3}, \frac{7}{8}, \frac{6}{15} =$

(b)

$\frac{2}{3}, \frac{4}{7}, \frac{4}{5} =$

10. Convert into improper fractions.

(a)

$4\frac{2}{3} =$

(b)

$8\frac{1}{2} =$

(c)

$13\frac{2}{9} =$

11. Convert into mixed fractions.

(a)

$\frac{118}{7}$

(b)

$\frac{2x^2-5}{x+2}$

(c)

$\frac{49}{4}$

Check your answers with those on the next page. Review those items that you missed before continuing to the next lesson in this study unit.

Lesson 1 Exercise Solutions

		References
1.	Positive	2101
2.	Integer	2101
3.	Integer Factor	2101
4.		
	(a) $6x^2y^2$	2102
	(b) $2xy$	2102
5.		
	(a) $b^2(9a^3+6a^2+2b^2)$	2102
	(b) Prime Polynomial	2102
	(c) $x^2(6+7x)$	2102
6.		
	(a) $\left(9x+9y\right)^2$	2103
	(b) $\left(x+y\right)^2$	2103
	(c) $(5a+5b)(5a-5b)$	2103
7.		
	(a) $\left(3x+2y\right)\left(x+2y\right)$	2104
	(b) $\left(5x+2y\right)\left(2x+3y\right)$	2104
8.		
	(a) $\frac{a+b}{a-b}$	2105
	(b) $11xy^2z+15$	2105
9.		
	(a) $\frac{320}{120}, \frac{105}{120}, \frac{48}{120}$	2106
	(b) $\frac{70}{105}, \frac{60}{105}, \frac{84}{105}$	2106
10.		
	(a) $\frac{14}{3}$	2107
	(b) $\frac{97}{12}$	2107
	(c) $\frac{119}{9}$	2107
11.		
	(a) $16\frac{6}{7}$	2108
	(b) $2x-4+\frac{3}{x+2}$	2108
	(c) $12\frac{1}{4}$	2108

Lesson 2. ADDITION AND SUBTRACTION OF FRACTIONS

Introduction. In lesson 1 of this study unit, you were introduced to factoring integers and literal numbers. You will build on your foundation by finding the sum and difference of fractions. This is the next step you must perform before you begin finding the product of fractions, a topic you will cover in the next lesson of this study unit.

LEARNING OBJECTIVES

1. Compute the sum of given common fractions.

2. Compute the difference of given common fractions.

3. Compute the sum of given literal fractions, using the rules for adding literal fractions.

4. Compute the difference of given literal fractions, using the rules for subtracting literal fractions.

2201. Add Common Fractions

You are ready to add common fractions. To do so, you must have common/like denominators. The first step in this operation is to reduce all original fractions having common denominators. As you did in the last lesson (Study Unit 2, lesson 1) you must find the Lowest Common Multiple (LCM) which will be the Lowest Common Denominator (LCD). After doing so, you must combine the numerators and place them over the common denominator. Take the sum of the numerator and reduce the fraction to its lowest term. This may seem pretty easy, but when we start mixing literal numbers in, it can seem harder but it is the same thing. Study the examples we have provided for you.

Example:

Reduced fractions. $\frac{2}{4} + \frac{6}{10} + \frac{4}{6}$

$$\frac{2}{4} + \frac{6}{10} + \frac{4}{6} = \frac{1}{2} + \frac{3}{5} + \frac{2}{3}$$

Denominators	Prime factors
2	2
5	5
3	3

LCM = 2*5*3

LCD =30

w Step - Divide the LCM or LCD by the denominator and multiply both the numerator and denominator by this quotient.

$$\frac{30}{2} = 15, \ \frac{30}{5} = 6, \ \frac{30}{3} = 10$$

$$\frac{(1*15)}{(2*15)} + \frac{(3*6)}{(5*6)} + \frac{(2*10)}{(3*10)}$$

$$\frac{15}{30} + \frac{18}{30} + \frac{20}{30}$$

w Step - Place the numerators over the LCD and algebraically add.

$$\frac{15+18+20}{30} = \frac{53}{30}$$

Example:

$$\frac{1}{48} + \frac{1}{4} + \frac{4}{15}$$

<u>Note</u>: Fractions can't be reduced in this example.

Denominators	Prime factors
48	2*2*2*2*3
4	2*2
15	3*5

LCM = 2*2*2*2*3*5. LCD = 240

$$\frac{(1*5)}{(48*5)} + \frac{(1*60)}{(4*60)} + \frac{(4*16)}{(15*16)}$$
$$\frac{5}{240} + \frac{60}{240} + \frac{64}{240}$$

$$\frac{5+60+64}{240} = \frac{129}{240}$$

$$\frac{\cancel{3}*43}{2*2*2*2*\cancel{3}*5} = \frac{43}{80}$$

Try the following challenge to give you practice and to reinforce your acquired knowledge on adding fractions.

(Add the following fractions)

(a)

$$\frac{3}{5} + \frac{4}{6} + \frac{2}{3} =$$

(b)

$$\frac{1}{2} + \frac{3}{8} + \frac{1}{3} =$$

Your answers in the challenge should match those that follow. If your answers are correct, continue. If your answers are incorrect, check your work for mathematical mistakes and review paragraph 2201 to ensure you understand adding fractions before continuing.

(Solutions)

(a)

$$\frac{3}{5} + \frac{4}{6} + \frac{2}{3} =$$

$$\frac{3}{5} + \frac{2}{3} + \frac{2}{3} =$$

$$\frac{9}{15} + \frac{10}{15} + \frac{10}{15} = \frac{2}{1}$$

(b)

$$\frac{1}{2} + \frac{3}{8} + \frac{1}{3} =$$

$$\frac{12}{24} + \frac{9}{24} + \frac{8}{24} = \frac{2}{2}$$

Now that you have finished adding common fractions, you are ready for the inverse operation of subtracting common fractions. Since subtracting is nothing more than adding with negative numbers, you will go through the same procedure as you did in the last paragraph. The fractions must have common/like denominators. The first step in this operation is to reduce all original fractions where possible. Then convert them to equivalent fractions having a common denominator. You will need to find the LCM which will be the LCD. Once you have the LCD, combine the numerators and place them over the common denominators. Take the algebraic sum of the numerators and perform the last step which is to reduce the fraction to its lowest terms. Study the examples we provided.

Example:

$$\frac{13}{42} - \frac{19}{36}$$

LCM = 2 * 2 * 3 * 3 * 7

LCD = 252

w Step - Find the equivalent fractions.

$$\frac{13}{42} = \frac{(13*6)}{(42*6)} = \frac{78}{252}$$

$$\frac{19}{36} = \frac{(19*7)}{(36*7)} = \frac{133}{252}$$

$$\frac{78}{252} - \frac{133}{252} =$$

w Step - Combine the numerators and place over the common denominator.

$$\frac{78-133}{252} = \frac{-55}{252}$$

Note: Remember, there are three signs to a fraction: the fraction, the numerator, and the denominator. In the solution above, the sign of the fraction is negative.

Try the following challenge to reinforce your knowledge of subtracting fractions.

(Subtract the following fractions)

(a)

$\frac{15}{16} - \frac{3}{8} - \frac{1}{2} =$

(b)

$\frac{7}{8} - \frac{12}{24} - \frac{1}{3} =$

Your answers in the challenge should match those below. If your answers are correct, continue. If they are not, before continuing, check your work for mathematical mistakes and review paragraph 2202 to be sure that you understand the material.

(Solutions)

(a)

$\frac{15}{16} - \frac{3}{8} - \frac{1}{2} =$

$\frac{15}{16} - \frac{6}{16} - \frac{8}{16} =$ [illegible]

(b)

$\frac{7}{8} - \frac{12}{24} - \frac{1}{3} =$

$\frac{21}{24} - \frac{12}{24} - \frac{8}{24} =$ [illegible]

2203. Adding Literal Fractions

Now that you have finished both adding and subtracting common fractions, you are ready to incorporate literal numbers in your fractions. The fractions must have common/like denominators. The first step in this operation is to reduce all original fractions where possible. Then, convert them to equivalent fractions having a common denominator. You will need to find the LCM which will be the LCD. Once you have the LCD, combine the numerators and place them over the common denominator. Add and reduce the fraction to its lowest terms. Study the examples provided on the next page.

Example:

$$5 + \frac{5x-30}{x^2-2x}$$

$$5 + \frac{5(x-6)}{x(x-2)}$$

Note: There are no terms that can be factored out, so it cannot be reduced.

LCM = 1 * $x(x - 2)$

LCD = $x(x - 2)$

w Step - Convert to equivalent fractions.

$$5 + \frac{5x-30}{x^2-2x}$$

$$= \frac{5\left(x^2-2x\right)}{1(x^2-2x)} + \frac{5x-30}{x^2-2x}$$

$$= \frac{5x^2-10x}{x^2-2x} + \frac{5x-30}{x^2-2x}$$

$$= \frac{5x^2-10x+5x-30}{x^2-2x}$$

$$= \frac{5x^2-5x-30}{x^2-2x}$$

w Step - Reduce if possible. Remember to factor.

$$= \frac{5(x^2-x-6)}{x(x-2)}$$

$$= \frac{5(x-3)(x+2)}{x(x-2)}$$

Example:

$$\frac{2IR}{8} + \frac{3IR}{6}$$

$$\frac{\cancel{2} * I * R}{\cancel{2} * 2 * 2} + \frac{\cancel{3} * I * R}{2 * \cancel{3}}$$

$$= \frac{IR}{4} + \frac{IR}{2}$$

LCM = 2 * 2

LCD = 4

w Step - Convert to equivalent fractions.

$$= \frac{IR}{4} + \frac{2(IR)}{2*2}$$

$$= \frac{IR}{4} + \frac{2IR}{4}$$

$$= \frac{IR+2IR}{4}$$

$$= \frac{3IR}{4}$$

2204. Subtract Literal Fractions

The final operation you are going to cover in this lesson is subtracting literal fractions. Just as you have done in the last three lessons, you must make sure the fractions have common/like denominators. Make sure the original fractions are reduced where possible. Convert them to equivalent fractions having a common denominator. You will again obtain the LCM which will be the LCD. Once you have the LCD, combine the numerators and place them over the common denominator. Now you are ready to algebraically add, watching the signs of the subtrahend and assigning the appropriate sign to the larger number. Reduce the fraction to its lowest terms. Study the examples that follow:

Example:

(a)

$$\frac{2E}{5} - \frac{3E}{6}$$

$$\frac{2E}{5} - \frac{3E}{6} = \frac{2E}{5} - \frac{E}{2}$$

LCM = 5 * 2

LCD = 10

w Step - Convert to equivalent fractions.

$$= \frac{2(2E)}{5*2} - \frac{5(E)}{2*5}$$

$$= \frac{4E}{10} - \frac{5(E)}{10}$$

$$= \frac{4E-5E}{10}$$

$$= \frac{-E}{10}$$

Example:

(b)

$$\frac{-17e}{2} - \frac{-13e}{6} - \frac{12e}{4}$$

$$\frac{-17e}{2} - \frac{-13e}{6} - 3e$$

LCM = 6

$$= \frac{3(-17e)}{3*2} - \frac{(-13e)}{6} - \frac{6(3e)}{6*1}$$

$$= \frac{-51e}{6} - \frac{-13e}{6} - \frac{18e}{6}$$

$$= \frac{(-51e)-(-13e)-(18e)}{6}$$

$$= \frac{-56e}{6}$$

w Step - Reduce if possible by factoring.

$$= \frac{-\cancel{2} * 2 * 2 * 7 * e}{\cancel{2} * 3}$$

$$= \frac{-28e}{3}$$

Now that you have studied the examples provided, try the following challenge.

(Perform the indicated operation)

(a)

$$\frac{9x}{4x^2-2x-2} - \frac{3}{x-1} =$$

(b)

$$\frac{2x}{y} + \frac{3y}{x} =$$

Your answer to the challenge should match the following. If your answers are correct, continue. If they are not, check your work for mathematical mistakes and review paragraph 2203 to be sure that you understand before continuing.

(Solutions)

(a)

$$\frac{9x}{4x^2-2x-2} - \frac{3}{x-1} =$$

$$\frac{9x}{(4x+2)(x-1)} - \frac{3}{x-1} =$$

$$\frac{9x-3(4x+2)}{(4x+2)(x-1)} =$$

$$\frac{9x-12x-6}{4x^2-2x-2} =$$

$$\frac{-3x-6}{4x^2-2x-2}$$

(b)

$$\frac{2x}{y} + \frac{3y}{x} =$$

$$\frac{2x^2}{xy} + \frac{3y^2}{xy} =$$

$$\frac{2x^2+3y^2}{xy}$$

Lesson Summary. You have completed lesson 2 of Study Unit 2. In this lesson, you learned to process the sum and difference of common fractions. You performed these processes with both whole and literal numbers. You learned to convert multiple fractions into equivalent fractions having a common denominator by finding that the Lowest Common Multiple (LCM) of the original denominators was the Lowest Common Denominator (LCD). The lesson exercise will serve as a measuring device to determine how well you learned the objectives of this lesson. It will also point out those items that you should review before taking the study unit exercise. Concentrate on each item carefully.

Lesson 2 Exercise: Complete items 1 through 4 by performing the action required. Check your responses against those listed at the end of this lesson.

1. Add the following fractions.

(a) $\frac{1}{3} + \frac{3}{4} + \frac{2}{7} =$

(b) $\frac{3}{5} + \frac{6}{18} + \frac{1}{4}$

2. Subtract the following fractions.

(a) $\frac{9}{8} - \frac{7}{24} - \frac{2}{3} =$

(b) $\frac{55}{64} - \frac{5}{8} - \frac{7}{32}$

3. Add the literal fraction.

(a)

$\frac{a}{b} + \frac{c}{d} =$

(b)

$\frac{2IR}{7} + \frac{3IR}{9} =$

4. Subtract the literal fraction.

(a)

$\frac{3x}{2y} - \frac{5y}{7x}$

(b)

$\frac{7x}{2x^2+x-1} - \frac{3}{x+1} =$

Check your answers with those on the next page. Review those items that you missed before continuing to the next lesson in this study unit.

Lesson 2 Exercise Solutions

		References
1.		
	(a) $1\frac{31}{84}$	2201
	(b) $1\frac{11}{60}$	2201
2.		
	(a) $\frac{1}{6}$	2202
	(b) $\frac{1}{64}$	2202
3.		
	(a) $\frac{ad+bc}{bd}$	2203
	(b) $\frac{13IR}{21}$	2203
4.		
	(a) $\frac{21x^2-10y^2}{14xy}$	2204
	(b) $\frac{x+3}{2x^2+x-1}$	2204

Lesson 3. MULTIPLICATION AND DIVISION OF FRACTIONS

Introduction. In lessons 1 and 2 of this study unit, you were introduced to factoring integers and literal numbers. You built on your foundation by finding the sum and difference of factors. In this lesson, you will expand on your knowledge by multiplying and dividing fractions. You will find that most electronic formulas will require you to find a product or quotient of both integers and literal numbers.

LEARNING OBJECTIVES

1. Compute the product of given common fractions using the rules for multiplication of fractions.

2. Compute the product of given literal fractions using the rules for multiplication of literal fractions.

3. Solve for the quotient of given common fractions using the rules for division of fractions.

4. Solve for the quotient of given literal fractions using the rules for division of literal fractions.

2301. Multiplying Fractions

The next operation you will cover is multiplying fractions. The product of two or more fractions is the product of the numerators divided by the product of the denominators. Once you have multiplied the numerators and the denominators, you then reduce or factor where possible. You will see as you go through this paragraph that integers are easy. Study the examples we have provided you.

Example:

$$\left(\frac{-2}{3}\right)\left(\frac{4}{5}\right)$$

$$= \frac{-8}{15}, or -\frac{8}{15}$$

$$\left(\frac{3}{4}\right)\left(\frac{2}{3}\right)\left(\frac{5}{6}\right) = \frac{30}{72}$$

w Step - Reduce by factoring primes.

$$= \frac{\cancel{2} * \cancel{3} * 5}{\cancel{2} * 2 * 2 * \cancel{3} * 3}$$

$$= \frac{5}{12}$$

<u>Note</u>: To multiply a whole number and a fraction, multiply the numerator by the whole number and place the product over the denominator.

Example:

$$\left(\frac{3}{5}\right) * 4$$

$$= \left(\frac{3}{5}\right)\left(\frac{4}{1}\right)$$

$$= \frac{12}{5}$$

Example:

$$\left(1\frac{3}{4}\right)\left(5\frac{3}{10}\right)$$

$$= \left(\frac{4}{4} + \frac{3}{4}\right)\left(\frac{50}{10} + \frac{3}{10}\right)$$

$$= \left(\frac{7}{4}\right)\left(\frac{53}{10}\right)$$

$$= \frac{371}{40}$$

<u>Note</u>: When a factor occurs one or more times in any numerator or denominator, when multiplying, it may be dropped out (canceled).

Example:

$$\left(\frac{3}{4}\right)\left(\frac{7}{9}\right)$$

$$= \left(\frac{\cancel{3}}{2 * 2}\right)\left(\frac{7}{\cancel{3} * 3}\right)$$

$$= \frac{7}{12}$$

Note: The rules for multiplication of signed numbers still hold true.

Example:

$$\left(\frac{-3}{4}\right)\left(\frac{4}{-9}\right)\left(\frac{6}{12}\right)$$

$$= \frac{-1}{-6}$$

$$= \frac{1}{6}$$

Now that you have finished studying the examples provided, try the following challenge.

(Multiply the following fractions)

(a)

$$\frac{1}{2} \ast \frac{4}{5} \ast \frac{15}{24} =$$

(b)

$$\frac{3}{8} \ast \frac{6}{9} \ast \frac{1}{3} =$$

Your answers to the challenge should match those that follow. If your answers are correct, continue. If they do not match, check your work for mathematical mistakes and review paragraph 2301 to be sure you understand the steps you must perform when multiplying fractions before continuing.

(a)

$$\frac{1}{2} \ast \frac{4}{5} \ast \frac{15}{24} :$$

$$\frac{1}{1} \ast \frac{1}{1} \ast \frac{3}{12} : \frac{3}{12} = \frac{1}{4}$$

(b)

$$\frac{3}{8} \ast \frac{6}{9} \ast \frac{1}{3} :$$

$$\frac{1}{4} \ast \frac{1}{3} \ast \frac{1}{1} : \frac{1}{12}$$

2302. Multiply Literal Fractions

To multiply literal fractions you must factor the numerators and denominators. Cancel like factors in any numerator and denominator only as many times as each factor occurs in both the numerator and denominator. Finally, multiply numerator times numerator and denominator times denominator.

Example:

$$\left(\frac{2a^2-ab-b^2}{a^2+2ab+b^2}\right)\left(\frac{a^2-b^2}{4a^2+4ab+b^2}\right)$$

$$\frac{\cancel{(2a+b)}\,(a-b)}{(a+b)\,\cancel{(a+b)}}\quad\frac{\cancel{(a+b)}\,(a-b)}{(2a+b)\,\cancel{(2a+b)}}$$

$$=\left(\frac{a-b}{a+b}\right)\left(\frac{a-b}{2a+b}\right)$$

$$=\frac{a^2-2ab+b^2}{2a^2+3ab+b^2}$$

Try the following challenge to reinforce your knowledge on multiplying fractions with literal numbers.

(Multiply the following fractions)

(a)

$$\frac{a^2+2ab+b^2}{(a+b)}\bullet\frac{a-b}{4}=$$

(b)

$$\frac{x^2+3x+2}{x+1}\bullet\frac{x-5}{3x+6}=$$

2302 before continuing.

(a)

$$\frac{a^2+2ab+b^2}{(a+b)} \bullet \frac{a-b}{4} =$$

$$\frac{(a+b)(a+b)}{a+b} \bullet \frac{a-l}{4}$$

$$\frac{a+b}{1} \bullet \frac{a-b}{4} =$$

$$\frac{a^2-b^2}{4}$$

(b)

$$\frac{x^2+3x+2}{x+1} \bullet \frac{x-5}{3x+6}$$

$$\frac{(x+1)(x+2)}{(x+1)} \bullet \frac{x-5}{3(x+2)} = \frac{x-5}{3}$$

Why is this true?

Note that $\frac{(x+1)(x+2)}{(x+1)(x+2)} = 1$

You are ready to divide fractions. You will use the same method to divide fractions in algebra as you used in basic arithmetic. You must divide a fraction by inverting the divisor and performing a multiplication operation as you did in the preceding paragraph. In the following sample, all parts of the division equation are labeled to provide a clearer meaning of each part and how it is used when dividing fractions.

Sample:

$$\frac{(\textit{Dividend})\ \frac{5}{2} \leftarrow (\textit{Minor Vinculum})}{(\textit{Divisor})\ \frac{2}{3}} \leftarrow \ (\textit{Major Vinculum})$$

Examples:

$$\frac{\frac{5}{4}}{\frac{3}{6}}$$

$$= \left(\frac{5}{4}\right)\left(\frac{6}{3}\right)$$

$$= \frac{5}{2}$$

$$\left(\frac{5}{(2*2)}\right)\left(\frac{(2*3)}{3}\right)$$

<u>Note</u>: To divide one con ner, reduce both the numerator and the denominator to si ns.

$$\frac{\frac{1}{3}+\frac{1}{5}}{4-1}$$

$$\frac{\frac{5}{15}+\frac{3}{15}}{20-1}$$

$$\frac{\frac{5+3}{15}}{20-1}$$

$$\frac{\frac{8}{15}}{19}$$

$$\left(\frac{8}{15}\right)\left(\frac{5}{19}\right)$$

$$\left(\frac{8}{3}\right)\left(\frac{1}{19}\right)$$

$$=\frac{8}{57}$$

Try this challenge by dividing these fractions.

(Divide)

(a) $\frac{3}{8} \div \frac{3}{4} =$

(b) $\frac{12}{15} \div \frac{3}{5} =$

(a)

$$\frac{3}{8} \div \frac{3}{4} =$$

$$\frac{3}{8} * \frac{4}{3} =$$

$$\frac{1}{2} * \frac{1}{1} = \frac{1}{2}$$

(b)

$$\frac{12}{15} \div \frac{3}{5} =$$

$$\frac{12}{15} * \frac{5}{3} =$$

$$\frac{4}{3} * \frac{1}{1} = \frac{4}{3}$$

preceding paragraphs of this study unit. You will have a chance after the examples to use most of what you have learned about fractions to this point. We will also provide you with complex literal fractions to exercise your knowledge. Study the example.

Example:

$$\frac{\frac{ab^2}{xy}}{\frac{a^2b}{xy^2}}$$

$$= \left(\frac{ab^2}{xy}\right)\left(\frac{xy^2}{a^2b}\right)$$

$$= \frac{by}{a}$$

<u>Note</u>: The first step to complex literal fractions is to change both numerator and denominator to simple fractions. Then, you can perform the division by inverting and multiplying.

Example:

$$\frac{5a-\frac{1}{a+1}}{3+\frac{2}{a+1}}$$

$$=\frac{\frac{5a(a+1)-1}{a+1}}{\frac{3(a+1)+2}{a+1}}$$

$$=\frac{5a^2+5a-1}{3a+3+2}$$

$$=\frac{5a^2+5a-1}{3a+5}$$

Try the following challenge dividing literal fractions.

(Divide)

(a)

$$\frac{12}{x^2+2x+1}\div\frac{4}{x+1}=$$

(b)

$$\frac{a^2-b^2}{a^2+2ab+b^2}\div\frac{a-b}{a+b}=$$

Your answers to the challenge should match those that follow. If your answers to the equations are correct, you may continue. If not, check your work for mathematical mistakes and repeat paragraph 2304 to be sure you understand dividing literal fractions before continuing.

(a)

$$\frac{12}{x^2+2x+1} \div \frac{4}{x+1} =$$

$$\frac{12}{(x+1)(x+1)} * \frac{x+1}{4} =$$

$$\frac{3}{x+1} * \frac{1}{1} =$$

$$\frac{3}{x+1}$$

(b)

$$\frac{a^2-b^2}{a^2+2ab+b^2} \div \frac{a-b}{a+b} =$$

$$\frac{(a+b)(a-b)}{(a+b)(a+b)} * \frac{a+b}{a-b} =$$

$$\frac{a+b}{a+b} = 1$$

<u>Lesson Summary</u>. In this lesson, you learned to factor numbers and algebraic terms. You have learned to add, subtract, multiply, and divide fractions. You are ready for the lesson exercise.

Lesson 3 Exercise: Complete items 1 through 4 by performing the action required. Check your responses against those listed at the end of this lesson.

1. Multiply the following fractions.

(a)

$$\frac{2}{7} * \frac{9}{6} * \frac{3}{2} =$$

(b)

$$\frac{1}{3} * \frac{2}{7} * \frac{24}{15} =$$

2. Multiply the following literal fractions.

(a)

$$\frac{a^3+3a^2b+3ab^2+b^3}{a+b} \bullet \frac{a-b}{2c+3d} \bullet 3d =$$

(b)

$$\frac{x^2+15x+56}{x+1} \bullet \frac{x+1}{3x+24} =$$

3. Divide these fractions.

(a)

$\frac{16}{14} \div \frac{8}{28} =$

(b)

$\frac{9}{31} \div \frac{3}{69} =$

4. Divide these literal fractions.

(a)

$\frac{18}{x^2+10x+9} \div \frac{4}{x+9} =$

(b)

$\frac{a^2+b^2}{a^2+2ab+b^2} \div \frac{a-b}{a+b} =$

Check your answers with those on the next page. Review those items that you missed before continuing on to lesson 1 in study unit 3.

Lesson 3 Exercise Solutions

			Reference
1.			
	(a)	$\frac{9}{14}$	2301
	(b)	$\frac{16}{105}$	2301
2.			
	(a)	$\frac{3d(a+b)(a-b)}{2c+3d}$	2302
	(b)	$\frac{x+7}{3}$	2302
3.			
	(a)	4	2303
	(b)	$6\frac{21}{31}$	2303
4.			
	(a)	$\frac{18}{4(x+1)} = \frac{4.5}{(x+1)}$	2304
	(b)	$\frac{a^2+b^2}{a^2-b^2}$	2304

UNIT SUMMARY

In this study unit, you applied basic mathematical operations learned in Study Unit 1 to factoring and fractions. The study unit provided you with terms associated with operation of fractions. You covered mixed factoring, addition, subtraction, multiplication, and division of fractions. Study Unit 3 will allow you to apply the knowledge you have learned to electronic equations. Before continuing to Study Unit 3, complete the Study Unit 2 exercise.

Study Unit 2 Exercise: Complete items 1 through 20 by performing the action required. Check your responses against those listed at the end of this study unit exercise.

1. Factor. $2a^2 + 5ab + 2b^2$

a. $(2a+b)(a+2b)$

b. $(2a-b)(a+2b)$

c. $(2a-b)(a-2b)$

d. $(2a+2b)(a-b)$

2. Factor. $a^2 - b^2$

a. $(a+b)(a+b)$

b. $(a-b)(a-b)$

c. $(a-b)(a+b)$

d. $(b-a)(b+a)$

3. Add. $\frac{7}{16} + \frac{3}{32} + \frac{3}{8} =$

a. $\frac{13}{11}$

b. $\frac{29}{32}$

c. $\frac{31}{32}$

d. $\frac{27}{32}$

4. Subtract. $\frac{7}{8} - \frac{3}{5} - \frac{1}{2} =$

a. $\frac{9}{40}$

b. $-\frac{7}{40}$

c. $-\frac{9}{40}$

d. $\frac{7}{40}$

5. Divide. $\frac{\frac{2a}{4}}{2}$

a. 1

b. $\frac{a}{2}$

c. $\frac{4}{a^2}$

d. $\frac{a^2}{4}$

6. Multiply. $\frac{2}{3} * \frac{5}{8} * \frac{7}{10}$

a. $\frac{7}{24}$

b. $\frac{14}{48}$

c. $\frac{35}{120}$

d. $\frac{9}{24}$

7. Divide. $\frac{a+4}{a^2-b^2} \div \frac{a^2-16}{a^2-2ab+b^2}$

a. $\frac{a+b}{(a+b)(a-4)}$

b. $\frac{a+b}{(a-b)(a-b)}$

c. $\frac{a-b}{(a+b)(a-4)}$

d. $\frac{a+b}{(a-b)(a-4)}$

8. Factor. $25a^2 + 10a - 24$

a. $(5a+4)(5a-6)$

b. $(5a-4)(5a+6)$

c. $(5a+4)(5a+6)$

d. $(5a-4)(5a-6)$

9. Simplify. $x + y - \frac{x^2-y^2}{x-y} + 1$

a. 1

b. $x - y$

c. $\frac{x}{x-y}$

d. $\frac{y}{x-y}$

10. Divide. $\frac{20x^3y^2}{2xy} \div \frac{5x^2y}{x}$

a. $2x$

b. xy

c. $2xy$

d. $2y$

11. Add. $\frac{17}{51}+\frac{18}{72}+\frac{2}{12}$

a. $\frac{37}{72}$

b. $\frac{5}{6}$

c. $\frac{11}{12}$

d. $\frac{3}{4}$

12. Factor. $w^2+4yw+4y^2$

a. $(w-2y)(w-2y)$

b. $(w+2y)(w+2y)$

c. $(w-2y)(w+2y)$

d. $(w+4y)(w+y)$

13. Subtract. $\frac{60e}{30}-\frac{10e}{15}-\frac{2e}{3}$

a. $\frac{e}{2}$

b. $\frac{21e}{30}$

c. $\frac{2e}{3}$

d. $9a^6b^9c^3$

14. Simplify. $\frac{\frac{x+1}{x-1}}{\frac{x+1}{x^2-1}}$

a. $x-1$

b. $x+1$

c. $\frac{1}{x-1}$

d. $\frac{1}{x+1}$

15. Subtract. $\frac{4}{e-3}-\frac{2}{e+2}$

a. $\frac{2e+2}{e^2+e-6}$

b. $\frac{2e-2}{e^2-e-6}$

c. $\frac{2e-14}{e^2-e-6}$

d. $\frac{2e+14}{e^2-e-6}$

16. Simplify. $\frac{\frac{3}{5}-3}{2-\frac{2}{5}}$

a. $\frac{3}{2}$

c. $\frac{2}{3}$

b. $-\frac{3}{2}$

d. $-\frac{2}{3}$

17. Factor. $18m^4 + 12m^2 - 16$

a. $(6m^2 - 4)(3m^2 + 4)$

c. $(6m^2 - 4)(3m^2 - 4)$

b. $(6m^2 + 4)(3m^2 - 4)$

d. $(6m^2 + 4)(3m^2 + 4)$

18. Multiply. $\frac{4d}{2e} * \frac{6e}{12} * \frac{2}{3d}$

a. $\frac{3}{2}$

c. $\frac{2}{3}$

b. $\frac{2e}{3d}$

d. $\frac{3d}{2e}$

19. Simplify. $\frac{2-\frac{a+b}{a-b}}{2+\frac{a+b}{a-b}}$

a. $\frac{a-3b}{3a-b}$

c. $\frac{a-b}{a+b}$

b. $\frac{a+3b}{3a+b}$

d. $\frac{a+b}{a-b}$

20. Simplify. $\dfrac{1}{e-\dfrac{e^2-1}{e+\dfrac{1}{e-1}}}$

a. $\dfrac{e^2+e+1}{2e+1}$

b. $\dfrac{2e-1}{e^2-e+1}$

c. $\dfrac{2e+1}{e^2+e+1}$

d. $\dfrac{e^2-e+1}{2e-1}$

Check your answers with those on the next page. Review those items that you missed before continuing to the next study unit.

Study Unit 2 Exercise Solutions

		Reference
1.	a.	1404
2.	c.	1404
3.	b.	2203
4.	c.	2204
5.	d.	2304
6.	a.	2303
7.	c.	2304
8.	b.	1404
9.	a.	2104
10.	a.	2304
11.	d.	2203
12.	b.	1404
13.	c.	2204
14.	b.	2304
15.	d.	2204
16.	b.	2304
17.	a.	1404
18.	c.	2303
19.	a.	2304
20.	d.	2304

STUDY UNIT 3

EQUATIONS

Introduction. You have completed the first two study units of this course. You will use your acquired knowledge by applying it appropriately to equations frequently used in the electronic maintenance field. There are three lessons in this study unit. They cover simple equations, fractional equations, and linear equations. Upon completion of this study unit, you should have little or no difficulty in solving any electronic formula or equation that relates to electronic circuits.

Lesson 1. INTRODUCTION TO EQUATIONS

LEARNING OBJECTIVES

1. Demonstrate the seven algebraic axioms by applying them to given equations.

2. Solve given literal equations containing one unknown by applying appropriate axioms.

3. Solve given literal equations (formulas) for a specified variable by applying the appropriate axioms.

4. Solve given equations involving ratio and proportion.

3101. Solving Equations

For you to solve equations, you must understand certain facts about them regardless of whether they contain whole numbers or factions. The following material covers background information on the fundamentals of solving equations. The background material addresses definitions and information on axioms that you need. The seven axioms are discussed. You will be able to solve equations as you manipulate numbers across the equality sign and also maintain the truth of the equation. Study the following definitions. Examples are provided.

a. Equation. A mathematical statement that two numbers or quantities are equal. The equality sign (=) is used to separate the two statements

b. Left member. Terms to the left of the equality sign as shown in the following example.

c. Right member. Terms to the right of the equality sign as shown in the following example.

Example:

$$3E + 4 = 2E + 6$$

(Left member) = (Right member)

d. Identity (Identical Equation). An equation whose members are equal for all values that can be substituted for the literal contained in the equation.

Example:

$$4I(r + R) = 4Ir + 4IR$$

$$4Ir + 4IR = 4Ir + 4IR$$

Note: The equation is equal no matter what values are substituted for I, r, or R.

e. Conditional equation. An equation consisting of one or more literal numbers that cannot be satisfied for all values of real numbers. In the example below, only when the real number 4 is substituted for *e* can the equation be satisfied.

Example:

$$e + 3 = 7$$

$$e = 4$$

f. Solve an equation or find the root of the equation. Solving means to find the value or values of the unknown real number or numbers that will satisfy the equation.

Example:

$$i + 6 = 14$$

Note: The equation becomes an identity only when $i = 8$, therefore; 8 is the root of the equation.

g. Axiom. An axiom is truth or fact that is self-evident and requires no proof. Basically the seven axioms state that whatever is done to change the value of one side of an equation must be done to the other side (of an equal value), to maintain equality. The following are the seven axioms that are used when solving an equation.

(1) If the equal numbers are added to both sides of an equation, the sums are equal and the equality of the equation is maintained. Study the example below:

Example:

If $a = b$

Then $a + 2 = b + 2$

(2) If the equal numbers are subtracted from both sides of an equation, the remainders are equal and the equality of the equation is maintained. Study the example below:

Example:

If $a = b$

Then $a - 2 = b - 2$

(3) If both sides of an equation are multiplied by the same number, the products are equal and the equality of the equation is maintained. Study the example below:

Example:

If $a = b$

Then $a * 2 = b * 2$

(4) If both sides of an equation are divided by the same number, their quotients are equal and the equality of the equation is maintained. Study the example below:

Example:

If $a = b$

Then $\frac{a}{2} = \frac{b}{2}$

(5) If numbers that are equal to each other are equal to other numbers, they are all equal to each other. Study the example below.

Example:

If $a = 3$ and $b = 3$

Then $a = b$

(6) If both sides of an equation are raised to the same power, they are equal and maintain the equality of the equation. Study the example below:

Example:

If $a = b$

Then $a^2 = b^2$

(7) If you take roots of both sides of an equation, they are equal and maintain the equality of the equation. Study the example below:

Example:

If $a = b$

Then $\sqrt[n]{a} = \sqrt[n]{b}$

or

This can be shown as an equation with a fractional exponent:

$$a^{\frac{1}{n}} = b^{\frac{1}{n}}$$

3102. Using Axioms

a. Transpose or move equations. When using axioms you must remember the definitions. Remember they state that whatever change is made to one side, or member you must also do the same to the other side or member so that you maintain the integrity of the equation. In example (a) you will notice that a (y) was subtracted from both sides of the equation to isolate a specific term. Look at example (b) where the sign was changed and the term was moved from one side of the equation to the other. The results are the same.

Examples:

$x + y = 3$	$x + y = 3$
$x + y - y = 3 - y$ (Subtract y from both sides)	$x = 3 - y$
$x = 3 - y$ (Combine like terms)	

b. Reduce equations. The same method can be used to reduce an equation. In the examples that follow, you have an equation with the same terms with like signs that appear in both members of the equation. In example (b) when you find an equation with like terms and signs you can simply cancel these terms. Study the example.

Examples:

$3x + 2y = 3 + 2y$	$3x + 2y = 3 + 2y$
$3x + 2y - 2y = 3 + 2y - 2y$ (subtract)	$3x = 3$
$3x = 3$	

c. Change signs of equations. Sometimes while solving a problem you will find a need to change all the signs in an equation. The signs of all the terms of an equation may be changed without destroying the equality as in the examples below. This is nothing more than multiplying both sides by (-1). Study the example.

Examples:

$-a + 1 = -3$	$-a + 1 = -3$
$(-1)(-a + 1) = (-1)(-3)$	$a - 1 = 3$
$a - 1 = 3$	

d. Solve equations using axioms. The steps involved in using axioms to solve equations are to consolidate, or combine like terms whenever possible. Group all of the terms containing the unknown for which you are solving on the same side of the equation using the appropriate axioms. Solve for a single positive value of the required unknown term on one side of the equation. The unknown term for which you are solving may appear on only one side of the equation and by itself. Study the following example before trying the challenge in this paragraph.

Example:

$2x - 13 - 3x = 5x + 2 - x$

$2x - 3x - 13 = 5x - x + 2$ (combine like terms)

$-x - 13 = 4x + 2$

$-x - 4x = 2 + 13$ (grouping terms to one side of equation)

$-5x = 15$

$\frac{-5x}{-5} = \frac{15}{-5}$ (reduce by factoring)

$x = -3$ (solution)

Note: Check this solution by replacing the literal in the original equation with this solution. Without using the axioms, both sides of the equation must now reduce/simplify to exactly equal terms.

$2x - 13 - 3x = 5x + 2 - x$

$2 (-3) - 13 - 3 (-3) = 5 (-3) + 2 - (-3)$

$-6 -13 + 9 = -15 + 5$

$-10 = -10$

Try the following challenge to reinforce your knowledge. Substitute your solution for the literal to ensure your solutions are correct before you check your conclusions against the ones provided.

(Solve for the literal)

(a)

$$3x = 21 - 4x$$

(b)

$$9M - 3 = 5 + 5M$$

If your answer to the challenge is the same as follows you are correct and may continue. If your answer is incorrect, review paragraph 3102 before continuing.

(Solutions)

(a)

$$3x = 21 - 4x$$

$$3x + 4x = 21 - 4x + 4x$$

$$7x = 21$$

$$\frac{7x}{7} = \frac{21}{7}$$

$$x = 3$$

(b)

$$9M - 3 = 5 + 5M$$

$$9M - 5M - 3 + 3 = 5 + 3 + 5M - 5M$$

$$4M = 8$$

$$\frac{4M}{4} = \frac{8}{4}$$

$$M = 2$$

3103. Solve Formulas

Formulas are nothing more than equations that show a scientific relationship between some constants or variables. The procedures for solving formulas are the same as those for solving equations in the preceding paragraphs. You are now ready to use the seven axioms to isolate a required unknown in a given formula. In the first example, the electronic formula for resonance of a series circuit is used to help demonstrate procedures for isolating a specific variable. This will show you how to apply the axioms to isolate the variable for capacitance represented by the letter (C) in this formula. Study the example.

Example:

$f = \frac{1}{2\pi\sqrt{LC}}$, Solve for ($C$)

$f\ 2\pi\sqrt{LC} = 1$ (Multiply both sides by $2\pi\sqrt{LC}$)

$\sqrt{LC} = \frac{1}{2\pi f}$ (Divide both sides by $2\pi f$)

$LC = \frac{1}{4\pi^2 f^2}$ (Square both sides)

$C = \frac{1}{4\pi^2 f^2 L}$ (Multiply by $\frac{1}{L}$)

Try the following challenge. Substitute your solution for the literal to ensure your solutions are correct before you check your answers against those provided.

(Solve for the literal)

(a) $t = \frac{T(C-F)}{C}$ (Solve for F)

(b) $X_L = 2\pi FL$ (Solve for F)

If your answer to the challenge is the same as what follows, you are correct and may continue. If your answer is incorrect, review paragraph 3103 before continuing.

(Solutions)

(a)

$$t = \frac{T(C-F)}{C}$$

(Solve for F)

$$\left(\frac{C}{1}\right)t = \frac{T(C-F)}{C}\left(\frac{C}{1}\right)$$

$$tC = T(C-F)$$

$$tC = TC - TF$$

$$TF + tC = TC - TF + TF$$

$$TF + tC = TC$$

$$TF + tC - tC = TC - tC$$

$$TF = TC - tC$$

$$TF = C(T-t)$$

$$\frac{TF}{T} = \frac{C(T-t)}{T}$$

$$\frac{F}{1} = F = \frac{C(T-t)}{T}$$

(b)

$$x_L = 2\pi FL$$

(Solve for F)

$$\frac{x_L}{2\pi L} = \frac{2\pi I}{2\pi.}$$

$$\frac{x_L}{2\pi L} = \frac{2\pi I}{2\pi.}$$

$$\frac{x_L}{2\pi L} = \frac{2\pi LF}{2\pi L} = \frac{2\pi I}{2\pi L}F = 1F$$

$$F = \frac{x_L}{2\pi L}$$

3104. Ratios and Proportions

Frequently in the study of electronics, you will hear that some quantity is directly proportional to another quantity and indirectly proportional to still another. These types of equations are called ratios and proportions. In the equation Ohm's law $I = \frac{E}{R}$, the current (I) is directly proportional to the voltage (E) and inversely proportional to the resistance (R). Directly proportional means if you double E then I is also doubled. Inversely proportional means if you double R then I is halved. One of the areas where ratios frequently show up is in resistive bridge circuits. In the ratio example, we will find the value of R_x that will balance the equation. Study this example before performing the challenge.

Example:

$$\frac{R_1}{R_2} = \frac{R_x}{R_4}$$

$R_2 R_x = R_1 R_4$ (Cross multiplying)

$R_x = \frac{R_1 R_4}{R_2}$ (Dividing both members of the equation by R_2)

Try the following challenge to reinforce your knowledge. Check your work to ensure it is correct before checking your answers to the equations against those given.

(Solve for the literal indicated)

(a)

$k = \frac{M}{\sqrt{L_1 L_2}}$ (Solve for L_2)

(b)

$X_c = \frac{1}{2\pi FC}$ (Solve for C)

If your answer to the challenge is the same as what follows, you are correct and may continue. If your answer is incorrect, review paragraph 3104 before continuing.

(Solutions)

(a)

$$k = \frac{M}{\sqrt{L_1 L_2}} \quad \text{(Solve for } L_2\text{)}$$

$$k^2 = \frac{M^2}{\sqrt{L_1^2 L_2^2}}$$

$$k^2 = \frac{M^2}{L_1 L_2}$$

$$k^2 L_1 L_2 = \frac{M^2}{L_1 L_2} \bullet \frac{L_1 L_2}{1}$$

$$k^2 L_1 L_2 = M^2$$

$$\frac{L_1 L_2 k^2}{k^2 L_1} = \frac{M^2}{k^2 L_1}$$

$$L_2 = \frac{M^2}{k^2 L_1}$$

(b)

$$X_c = \frac{1}{2\pi FC} \text{(Solve for } C\text{)}$$

$$2\pi FCX_c = \frac{1}{2\pi FC} \bullet \frac{2\pi FC}{1}$$

$$X_c(2\pi FC) = 1$$

$$\frac{X_c 2\pi FC}{2\pi FX_c} = \frac{1}{2\pi FX_c}$$

$$C = \frac{1}{2\pi FX_c}$$

<u>Lesson Summary</u>. You have completed lesson 1 of Study Unit 3. In this lesson, you learned the seven axioms and their meanings. You applied the seven axioms to literal equations. You applied the appropriate axiom to an equation to solve for an unknown term. You can solve a literal equation or formula for a specific variable. You applied problem solving procedures to equations involving ratio and proportions. The lesson exercise will serve as a measuring device and it will also point out those items that you need to review before you take the study unit exercise. Concentrate on each item carefully.

Lesson 1 Exercise: Complete items 1 through 5 by performing the action required. Check your responses against those listed at the end of this lesson.

1. An _________ requires no proof.

   a. integer
   b. axiom
   c. integral
   d. equation

2. What do the seven axioms state?

____

____

____

3. Solve for the unknown literal number.

(a) $13M + 3 = 5 + 5M$

(b) $3x + (5 + x) = 47 - 8x$

4. Solve for the literal (F) in both of the following equations.

(a) $X = \frac{2\pi\sqrt{FL}}{rh}$

(b) $t = \frac{T\pi(C+F)}{C}$

5. Solve for the literal as indicated.

(a)

$X_c = \frac{xyz}{2\pi FC^2}$ (Solve for C)

(b)

$k = \frac{Mb+Nj}{a\sqrt{L_1 L_2}}$ (Solve for L_2)

Check your answers with those on the next page. Review those items that you missed before continuing on to the next lesson in this study unit.

Lesson 1 Exercise Solutions

		Reference
1.	b.	3101
2.	Your answer should be similar to the following: Whatever is done to one side, or member, of an equation must be done to the other side, or member, of the equation.	3101
3.		
	(a) $M = .25$	3102
	(b) $x = 3.5$	3102
4.		
	(a) $F = \frac{r^2h^2X^2}{4\pi^2L}$	3103
	(b) $F = \frac{C_t - T\pi C}{T\pi}$	3103
5.		
	(a) $C = \sqrt{\frac{xyz}{2\pi FX_c}}$	3104
	(b) $L_2 = \frac{\left(\frac{M_b+N_j}{ka}\right)^2}{L_1}$	3104

Lesson 2. FRACTIONAL EQUATIONS

Introduction. In lesson 1 of this study unit you learned the foundation for solving equations. In this lesson you will continue to build on your acquired knowledge. You will be working with fractional equations and equations with decimal coefficients. Upon completion of this lesson, you will be able to apply your acquired skills and knowledge in any given situation as required.

LEARNING OBJECTIVES

1. Solve given equations containing fractions by clearing the equations of fractions and applying the appropriate axioms.

2. Solve given equations containing decimal coefficients by clearing the equation of decimals and applying the appropriate axioms.

3201. Solve Fractional Equations with and without Coefficients

Fractional equations are a common occurrence in electronics. The mathematical problem-solving principles you have learned will be used and applied to given situations. You are given examples of equations demonstrating the two methods used in solving fractional equations. You should always remember that the first step in solving any fraction with fractional coefficients is to reduce them wherever possible. After reducing, you must find the Least Common Denominator (LCD). In example (a) you will see that a common denominator for both members of the equation is used. In example (b) each member has a different common denominator. The major difference in the methods is that in example (a) you must multiply both members by the common denominator, and in example (b) you must cross multiply to remove the denominators. Study each example carefully and determine which method is easier for you. Both methods will give you the same solution. Remember, a fractional equation is simply an equation that has a variable in at least one of the denominators.

a. <u>Solve fractional equations without coefficients</u>.

Example:

Common denominator used in both members of equation.

$$2x + \frac{x}{3} - 7 = \frac{3x}{2} - 4 - \frac{2x}{12}$$ (Reduce fractions)

$$2x + \frac{x}{3} - 7 = \frac{3x}{2} - 4 - \frac{x}{6}$$ (Determine LCD)

$$\frac{(6)(2x)}{(6)(1)} + \frac{(2)(x)}{(2)(3)} - \frac{(6)(7)}{(6)(1)} = \frac{(3)(3x)}{(3)(2)} - \frac{(6)(4)}{(6)(1)} - \frac{x}{6}$$ (Convert all fractions to LCD)

$$\frac{12x}{6} + \frac{2x}{6} - \frac{42}{6} = \frac{9x}{6} - \frac{24}{6} - \frac{x}{6}$$

$$\frac{12x+2x-42}{6} = \frac{9x-24-x}{6}$$ (Combine numerators over LCDs)

$$\frac{\cancel{6}}{1} * \frac{12x + 2x - 42}{\cancel{6}} = \frac{9x - 24 - x}{\cancel{6}} * \frac{\cancel{6}}{1}$$

(Multiply equation by LCD)

$$12x + 2x - 42 = 9x - 24 - x$$ (Combine like terms)

$$14x - 42 = 8x - 24$$ (Isolate the unknown)

$$14x - 8x = 42 - 24$$ (Combine like terms)

$$6x = 18$$ (Divide both members by 6)

$$\frac{6x}{6} = \frac{18}{6}$$

$$x = 3$$

<u>Note</u>: In the next example you will use an LCD for each member (not a common denominator for both members) and cross multiply to solve the equation.

Cross multiplication method for equations with fractional coefficients.

Example:

$2x + \frac{x}{3} - 7 = \frac{3x}{2} - 4 - \frac{2x}{12}$	(Reduce all fractions)
$2x + \frac{x}{3} - 7 = \frac{3x}{2} - 4 - \frac{x}{6}$	(Determine the LCDs)
$\frac{(3)(2x)}{(3)(1)} + \frac{x}{3} - \frac{(3)(7)}{(3)(1)} = \frac{(3)(3x)}{(3)(2)} - \frac{(6)(3x)}{(3)(2)} - \frac{x}{6}$	(Convert all fractions to LCDs)
$\frac{6x}{3} + \frac{x}{3} - \frac{21}{3} = \frac{9x}{6} - \frac{24}{6} - \frac{x}{6}$	(Combine numerators over LCDs)
$\frac{6x+x-21}{3} = \frac{9x-24-x}{6}$	(Cross multiply)
$(6) * (6x + x - 21) = (3) * (9x - 24 - x)$	(Combine where possible)
$(6) * (7x - 21) = (3) * (8x - 24)$	(Multiply)
$42x - 126 = 24x - 72$	(Isolate the unknown)
$42x - 24x = 126 - 72$	(Combine like terms)
$18x = 54$	(Result of the line above)
$\frac{18x}{18} = \frac{54}{18}$	(Divide both members by 18)
$x = 3$	

Note: You can see that we derived the same solution $x = 3$ using both methods. If we substitute the whole number 3 for the literal number x, we will prove the solutions.

$2x + \frac{x}{3} - 7 = \frac{3x}{2} - 4 - \frac{2x}{12}$ Solution (0 = 0)

$2(3) + \frac{(3)}{3} - 7 = \frac{3(3)}{2} - 4 - \frac{2(3)}{12}$

$6 + \frac{3}{3} - 7 = \frac{9}{2} - 4 - \frac{6}{12}$

$6 + 1 - 7 = \frac{9}{2} - 4 - \frac{1}{2}$

$7 - 7 = \frac{9-8-1}{2}$

b. Solve fractional equations with coefficients. Fractional equations differ from equations with fractional coefficients by containing a literal in the denominator.

$$\frac{x+2}{3x} - \frac{2x^2+3}{6x^2} = \frac{1}{2x}$$ (Fractions are not reducible)

$$\frac{x+2}{3x} - \frac{2x^2+3}{6x^2} = \frac{1}{2x}$$ (Determine LCD and convert all fractions)

$$\frac{2x(x+2)-(2x^2+3)}{6x^2} = \frac{3x(1)}{6x^2}$$ (Multiply numerators)

$$\frac{2x^2+4x-2x^2-3}{6x^2} = \frac{3x}{6x^2}$$ (Multiply equation by the LCD)

$$2x^2 + 4x - 2x^2 - 3 = 3x$$ (Use axioms to solve)

$$2x^2 - 2x^2 + 4x - 3x = 3$$ (Combine like terms)

$$x = 3$$

Replace the literal number "x" with the whole number 3 to prove the solution.

$$\frac{x+2}{3x} - \frac{2x^2+3}{6x^2} = \frac{1}{2x}$$ Solution $\left(\frac{9}{54} = \frac{9}{54}\right)$

$$\frac{(3)+2}{(3)3} - \frac{2(3)^2+3}{6(3)^2} = \frac{1}{2(3)}$$

$$\frac{(3)+2}{(3)3} - \frac{2(3)^2+3}{6(3)^2} = \frac{1}{2(3)}$$

$$\frac{5}{9} - \frac{2(9)+3}{6(9)} = \frac{1}{6}$$

$$\frac{5}{9} - \frac{18+3}{54} = \frac{1}{6}$$

$$\frac{5}{9} - \frac{21}{54} = \frac{1}{6}$$

$$\frac{(6)5}{(6)9} - \frac{21}{54} = \frac{(9)1}{(9)6}$$

$$\frac{30-21}{54} = \frac{9}{54}$$ $\frac{9}{54} = \frac{9}{54}$

Try the following challenge to reinforce your understanding of the examples you just studied. Substitute your solution for the literal to check your work. This will ensure your solutions are correct before you check them against those given.

(Solve for the literal)

(a)

$$\frac{5x-2}{3} - \frac{2x+1}{2} = \frac{x+2}{4}$$

(b)

$$\frac{x-2}{x} = \frac{3}{5}$$

If your answer to the challenge is the same as follows, you are correct and may continue. If your answer is incorrect, review paragraph 3201 before continuing.

(Solution)

(a)

$$\frac{5x-2}{3} - \frac{2x+1}{2} = \frac{x+2}{4}$$

$$\frac{20x-8-(12x+6)}{12} = \frac{3x+6}{12}$$

$$\frac{20x-8-12x-6}{12} = \frac{3x+6}{12}$$

$$\frac{8x-14}{12} = \frac{3x+6}{12}$$

$$\frac{12}{1} \ast \frac{8x-14}{12} = \frac{3x+6}{12} \ast 1$$

$$8x - 14 = 3x + 6$$

$$5x = 20$$

$$x = 4$$

(b)

$$\frac{x-2}{x} = \frac{3}{5}$$

$$\frac{5}{1} \ast \frac{x-2}{x} = \frac{3}{5} \ast$$

$$5x - 10 = 3x$$

$$2x = 10$$

$$x = 5$$

3202. Decimal Equations

To solve decimal coefficients you must multiply both members of the equations by a power of 10 that corresponds to the greatest number of decimals appearing in the denominator of a fraction in the equation. Multiply both the numerator and the denominator of that fraction by the power of 10 that will change all decimals to an integer and reduce where possible. Study the examples provided.

a. Solve decimal equations.

Example:

$0.75 - 0.7a = 0.26$

$(100)\ (0.75 - 0.7a\ ) = 0.26\ (100)$ (Multiply by 100)

$75 - 70a = 26$ (Isolate the unknown)

$- 70a = -49$ (Combine like terms)

$a = \frac{-49}{-70}$ (Combine like terms)

$a = 0.7$

b. Solve an equation with decimals in the denominator.

Example:

$$\frac{5m-1.33}{0.02} - \frac{m}{0.05} = 1083.5$$

$$\left(\frac{100}{100}\right)\left(\frac{5M-1.33}{0.02} - \frac{m}{0.05}\right) = (1)(1083.5)$$ (Since $\frac{100}{100}$ = 1 both sides are multiplied by 1**)**

$$\frac{500m-133}{2} - \frac{100m}{5} = 1083.5$$ (Determine the LCD of the equation)

$$\frac{500m-133}{2} - \frac{100m}{5} = 1083.5$$ (Convert all fractions to the LCD)

$$\frac{5(500m-133)}{5(2)} - \frac{2(100m)}{2(5)} = 1083.5$$

$$\frac{2500m-665-200m}{10} = 1083.5$$ (Multiply both members by the LCD)

$$2500m - 200m = 10835 + 665$$ (Combine like terms)

$$2300m = 11500$$

$$m = \frac{11500}{2300}$$

$$m = 5$$

c. <u>Prove the solution</u>. If we substitute the whole number 5 for the literal number "*m*" you will prove the solution.

Example:

$$\frac{5m-1.33}{0.02} - \frac{m}{0.05} = 1083.5$$

$$\frac{5(5)-1.33}{0.02} - \frac{(5)}{0.05} = 1083.5$$

$$\frac{25-1.33}{0.02} - \frac{5}{0.05} = 1083.5$$

$$\frac{23.67}{0.02} - \frac{5}{0.05} = 1083.5$$

$$1183.5 - 100 = 1083.5$$

$$1083.5 = 1083.5$$

Try the following challenge. Substitute your solutions for the literal to check your work to ensure your solutions are correct before checking them against those given.

If your answer to the challenge is the same as what follows, you are correct and may continue. If your answer is incorrect, review paragraph 3202 before continuing.

(Solve for the literal)

(a)

$$(.7a - .7)(.2 + a) = (1 - 1.4a)(.1 - .5a)$$

(b)

$$\frac{.4e-6}{.06e-.07} = \frac{2e-3}{.3e-.4}$$

(Solutions)

(a)

$$(.7a-.7)(.2+a)=(1-1.4a)(.1-.5a)$$

$$(7a-7)(2+10a)=(10-14a)(1-5a)$$

$$70a^2+14a-70a-14=70a^2-50a-14a+10$$

$$70a^2-56a-14=70a^2-64a+10$$

$$-56a-14=-64a+10$$

$$8a-14=10$$

$$8a=24$$

$$a=3$$

(b)

$$\frac{4e-6}{.6e-.7}=\frac{2e-3}{.3e-.4}$$

Multiply both sides of the equation by

$$\frac{10}{10}$$

$$\frac{40e-60}{6e-7}=\frac{20e-30}{3e-4}$$

$$(40e-60)(3e-4)=(6e-7)(20e-30)$$

$$120e^2-160e-180e+240=120e^2-180e-140e+210$$

$$120e^2-340e+240=120e^2-320e+210$$

$$-340e+240=-320e+210$$

$$240=20e+210$$

$$30=20e$$

$$e=\frac{30}{20}=\frac{3}{2}=1.5$$

<u>Lesson Summary</u>. You have completed lesson 2 of Study Unit 3. In this lesson, you learned about fractional equations and equations containing decimal coefficients. You learned two methods of solving fractional equations, cross multiplication method and factoring with a LCD.

You will be able to apply your acquired skill and knowledge on electronic formulas in any given situation as required.

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Lesson 2 Exercise: Complete items 1 through 4 by performing the action required. Check your responses against those listed at the end of this lesson.

1. Solve for the literal number *x*.

$$\frac{x+2}{x+9} = \frac{3}{5}$$

2. Solve for the literal number *x*.

$$\frac{7x+5}{3} + \frac{3x+2}{2}$$

$$= 4.5x$$

3. Solve for the literal number *x*.

$$3x + \frac{2x}{6} - 9 = \frac{6x}{3} - \frac{4x}{12}$$

4. Solve for the literal number *a*.

$$.95a + .3 - 0.33a = .22 + .3a$$

Check your answers with those on the next page. Review those items that you missed before continuing on to the next lesson in this study unit.

Lesson 2 Exercise Solutions

		Reference
1.	$x = 8.5$	3201
2.	$x=4$	3201
3.	$x = \frac{27}{5}$	3201
4.	$a = -.25$	3202

Lesson 3. SIMULTANEOUS LINEAR EQUATIONS

Introduction. In lessons 1 and 2 of this study unit, you learned the foundation for solving whole numeral and fractional equations. In this lesson, you will continue to build on your acquired knowledge. You will be working with simultaneous linear equations with multiple unknowns. Upon completion of this lesson, you will be able to apply your acquired skills and knowledge in any given situation as required.

LEARNING OBJECTIVES

1. Solve given simultaneous linear equations containing two unknowns using the elimination by addition/subtraction, substitution, and comparison methods.

2. Solve given simultaneous linear equations containing fractions.

3. Solve given simultaneous linear equations containing three unknowns using the elimination by addition/subtraction, and substitution methods.

4. Solve given simultaneous linear equations containing three unknowns using the determinants matrix.

3301. Simultaneous Linear Equations

Often in electronics you are faced with a circuit that will meet several conditions at the same time. The properties of the circuit can be found by applying the required mathematical operation for a common solution. Therefore, you must be able to compare such conditions through simultaneous equations. By definition, simultaneous linear equations are a set of equations that impose different conditions on the same variable and have a common solution. In this lesson, you will learn to solve such equations using the methods that follow. A brief definition and an explanation of the processes are also given.

a. Addition/subtraction method. The basic operation of adding and/or subtracting algebraic equations using the procedure(s) as required depending on the number of conditions in the equation.

Note: In this situation, one literal from two different equations is eliminated.

It may be necessary to multiply each equation by a number that will make the coefficients of one of the variables an absolute equal value.

Note: It does not matter which variable is selected.

In the example that follows, the variable "b" is eliminated by multiplying each formula respectively by the numbers 2 and 3 so that the coefficients of "b" equal 6.

After eliminating the coefficients of "b" the problem is solved for "a." The solution is substituted for the variable "a" and is solved for the number that satisfies both equations for the literal "b." Study the example.

Example:

$$3a + 3b = 15$$
$$5a - 2b = 4$$

$$(2)3a + (2)3b = (2)15 \qquad (*2)$$
$$(3)5a - (3)2b = (3)4 \qquad (*3)$$

$$6a + 6b = 30$$
$$15a - 6b = 12$$

If the coefficients of equal absolute value have like signs, subtract one equation from the other; if they have unlike signs, add the equations. In this example we will add because the coefficients have opposite signs.

Example:

$$\begin{array}{r} 6a + 6b = 30 \\ \underline{15a - 6b = 12} \\ 21a \qquad\quad 42 \end{array}$$

$$21a = 42$$
$$a = \tfrac{42}{21}$$
$$a = 2$$

Substitute the value "2" for the variable "a" in the original equations, and solve for the remaining variable.

Example:

$$3a + 3b = 15 \qquad \text{(original formula)}$$
$$3(2) + 3b = 15 \qquad \text{(2 substituted for variable a)}$$
$$6 + 3b = 15 \qquad \text{(subtract 6 from both members)}$$
$$6 - 6 + 3b = 15 - 6$$
$$3b = 9 \qquad \text{(factor for b)}$$
$$\frac{3b}{3} = \frac{9}{3}$$
$$b = 3$$

Check the solution by substituting the variables in the original equations with (a = 2) and (b = 3).

Example:

$$\begin{aligned} 3a + 3b &= 15 \\ 3(2) + 3(3) &= 15 \\ 6 + 9 &= 15 \\ 15 &= 15 \end{aligned} \qquad \begin{aligned} 5a - 2b &= 4 \\ 5(2) - 2(3) &= 4 \\ 10 - 6 &= 4 \\ 4 &= 4 \end{aligned}$$

The following challenge reinforces your knowledge. Substitute your solutions for the literal to make sure your solutions for each item are correct before checking your solutions against those given.

(Solve the linear equations using the addition/subtraction method)

(a)

$$2a + 7b = 31$$
$$6a - 3b = 21$$

(b)

$$3x + 2y = 13$$
$$4x - 7y = -2$$

If your answer to the challenge is the same as what follows, you are correct and may continue. If your answer is incorrect, review paragraph 3301 on solving simultaneous linear equations using the addition/subtraction method before continuing.

(Solution to linear equations using the addition/subtraction method)

(a)

$$2a + 7b = 31$$
$$6a - 3b = 21$$
$$(3)2a + (3)7b = (3)31$$
$$6a - 3b = 21$$
$$6a + 21b = 93$$
$$-6a + 3b = -21$$
$$24b = 72$$
$$\frac{24b}{24} = \frac{72}{24}$$
$$b = 3$$
$$2a + 7(3) = 31$$
$$2a + 21 = 31$$
$$2a + 21 - 21 = 31 - 21$$
$$2a = 10$$
$$\frac{2a}{2} = \frac{10}{2}$$
$$a = 5$$
$$(a = 5, b = 3)$$

(b)

$$3x + 2y = 13$$
$$4x - 7y = -2$$
$$(4)3x + (4)2y = (4)13$$
$$(3)4x - (3)7y = (3) - 2$$
$$12x + 8y = 52$$
$$\underline{-12x + 21y = 6}$$
$$29y = 58$$
$$\frac{29y}{29} = \frac{58}{29}$$
$$y = 2$$
$$3x + 2(2) = 13$$
$$3x + 4 = 13$$
$$3x + 4 - 4 = 13 - 4$$
$$3x = 9$$
$$\frac{3x}{3} = \frac{9}{3}$$
$$x = 3$$
$$(x = 3, y = 2)$$

b. Substitution method. The substitution method is an alternate algebraic method of solving simultaneous linear equations. The reasoning behind using this method is that the equations must be true at the same time. To use this method you do the following:

(1) Use one of the equations and solve for one of the variables in terms of the other variable.

(2) Take the results you derived from the first equation and substitute its value for the variable in the other equation.

(3) Make equivalent equations and combine as indicated.

(4) Substitute this for the variable for either of the original equations and solve for the other variable.

Note: Use the same formulas established in the addition/subtraction method to make it easier to see how the substitution method differs. Study the steps in the example that follows:

Example:

$$3a + 3b = 15$$
$$5a - 2b = 4$$

Use one equation and solve for one variable, in this case "a" in terms of the variable "b."

$$5a - 2b = 4$$
$$5a = 4 + 2b$$
$$a = \frac{4+2b}{5}$$ (Resultant value from one equation)

$$3a + 3b = 15$$ (Other original formula)

$$3\left(\frac{4+2b}{5}\right) + 3b = 15$$ (Resultant value substituted in other equation)

$$\frac{12+6b}{5} + 3b = 15$$

$$\frac{12+6b+15b}{5} = 15$$ (Equivalent fractions and combine like terms)

$$\frac{12+21b}{5} = 15$$

$$\frac{5}{1} * \frac{12+21b}{5} = 15 * 5$$ (Factor out the denominator)

$$12 + 21b = 75$$
$$12 - 12 + 21b = 75 - 12$$
$$21b = 63$$ (Factor for b)

$$\frac{21b}{21} = \frac{63}{21}$$

$$b = 3$$

Substitute the value "3" for the variable "b" in the original equations, and solve for the remaining variable.

$$3a + 3b = 15$$ (Either original formula)

$$3a + 3(3) = 15$$ (Substitute 3 for the literal b)

$$3a + 9 = 15$$

$$3a + 9 - 9 = 15 - 9$$

$$3a = 6$$

$$\frac{3a}{3} = \frac{6}{3}$$ (Factor for a)

$$a = 2$$

Solution: (*a* = 2, *b* = 3)

As you can see we got the same solution (*a* = 2, *b* = 3) using the substitution method as we did with the addition/subtraction method.

Use the substitution method to complete the following challenge. Substitute your solution for the literal to ensure your solutions to each item are correct before you check your solutions against those given.

(Solve the linear equations using the substitution method)

(a)

$$\begin{aligned} 8a + 3b &= 41 \\ a - 6b &= -14 \end{aligned}$$

(b)

$$\begin{aligned} 2x + 4y &= 24 \\ 7x - 4y &= -6 \end{aligned}$$

If your answer to the challenge agrees with what follows, you are correct and may continue. If your answer is incorrect, review paragraph 3301 on using the substitution method for solving simultaneous linear equations before continuing.

(Solution to the linear equations using the substitution method)

(a)

$$8a + 3b = 41$$
$$a - 6b = -14$$

$$a - 6b = -14$$
$$a = -14 + 6b$$

$$8a + 3b = 41$$
$$8(-14 + 6b) + 3b = 41$$
$$-112 + 48b + 3b = 41$$
$$-112 + 51b = 41$$
$$-112 + 112 + 51b = 41 + 112$$
$$51b = 153$$
$$\tfrac{51b}{51} = \tfrac{153}{51}$$
$$b = 3$$
$$a - 6b = -14$$
$$a - 6(3) = -14$$
$$a - 18 = -14$$
$$a - 18 + 18 = -14 + 18$$

$$a = 4$$

$$(a = 4, b = 3)$$

(b)

$$2x + 4y = 24$$
$$2x + 4y - 4y = 24 - 4y$$
$$2x = 24 - 4y$$
$$\tfrac{2x}{2} = \tfrac{24-4y}{2}$$
$$x = \tfrac{24-4y}{2}$$
$$7x - 4y = -6$$
$$7\left(\tfrac{24-4y}{2}\right) - 4y = -6$$
$$\tfrac{168-28y}{2} - 4y = -6$$
$$\tfrac{168-28y}{2} - \tfrac{8y}{2} = -6$$
$$\tfrac{168-28y-8y}{2} = -6$$
$$168 - 36y = -12$$
$$168 - 168 - 36y = -12 - 168$$

$$-36y = -180$$
$$\tfrac{-36y}{-36} = \tfrac{-180}{-36}$$
$$y = 5$$
$$2x + 4y = 24$$
$$2x + 4(5) = 24$$
$$2x + 20 = 24$$
$$2x + 20 - 20 = 24 - 20$$
$$2x = 4$$
$$\tfrac{2x}{2} = \tfrac{4}{2}$$
$$x = 2$$
$$(x = 2, y = 5)$$

c. Comparison method. Using this method, you solve both equations for the same variable in each equation in terms of the other variable. Then, you take the result that you derive from each equation and set the equations equal to each other.

Multiply using the axioms and factor for the literal. Substitute the variable into either one of the original equations and solve for the other variable. The same equation used in the previous methods is used in this example. Study the steps.

Note: Unlike the substitution method where you use one of the equations and solve for one of the variables in terms of the other variable, in the comparison method you use both of the equations and solve for the same variable in each equation in terms of the other variable. You then take the resultant value derived from each equation and set the equation equal to each other. Once you have done this, simply cross multiply and use your axioms factor for the literal. Substitute this variable into either one of the original equations and solve for the other variable. We will use the same formulas as we did before in the addition/subtraction and substitution methods to make it easier to see how the comparison method differs. Study the steps in the example that follows:

Example:

$$3a+3b=15$$
$$5a-2b=4$$

Factor for the literal "a" in both equations.

$$3a+3b=15$$
$$3a+3b-3b=15-3b$$
$$3a=15-3b$$
$$\frac{3a}{3}=\frac{15-3b}{3}$$
$$a=\frac{15-3b}{3}$$

$$5a-2b=4$$
$$5a-2b+2b=4+2b$$
$$5a=4+2b$$
$$\frac{5a}{5}=\frac{4+2b}{5}$$
$$a=\frac{4+2b}{5}$$

Note: Since ($a = a$) you can take the two results from the equations and make them equal.

$$\frac{15-3b}{3}=\frac{4+2b}{5}$$

Now cross multiply and solve for "b."

$$5(15-3b)=3(4+2b)$$
$$75-15b=12+6b$$
$$75-75-15b-6b=12-75+6b-6b$$
$$-15b-6b=12-75$$
$$-21b=-63$$
$$\frac{-21b}{-21}=\frac{-63}{-21}$$
$$b=3$$

Now substitute the value "3" of the variable "b" in either one of the original equations and solve for the remaining variable.

$$3a+3b=15$$
$$3a+3(3)=15$$
$$3a=6$$
$$\frac{3a}{3}=\frac{6}{3} \quad 3a+9=15$$
$$a=2 \quad 3a+9-9=15-9$$

As you can see, we got the same solution (*a* = 2, *b* = 3) using the comparison method as we did in the substitution method and the addition/subtraction method.

Try the following challenge to reinforce your knowledge. Substitute your solution for the literal to check your work before you check your results against those given.

(Solve the linear equations using the comparison method)

(a)

$$7a + 3b = 26$$
$$5a - 2b = 2$$

(b)

$$6x + y = 33$$
$$4x - 2y = 14$$

If your answer to the challenge is the same as follows, you are correct and may continue. If your answer is incorrect, review paragraph 3301 on using the comparison method for solving simultaneous linear equations before continuing.

(Solution to the linear equations using the comparison method)

(a)

$$7a + 3b = 26$$
$$5a - 2b = 2$$

$$7a + 3b = 26$$
$$7a + 3b - 3b = 26 - 3b$$
$$7a = 26 - 3b$$
$$\frac{7a}{7} = \frac{26-3b}{7}$$
$$a = \frac{26-3b}{7}$$

$$5a - 2b = 2$$
$$5a - 2b + 2b = 2 + 2b$$
$$5a = 2 + 2b$$
$$\frac{5a}{5} = \frac{2+2b}{5}$$
$$a = \frac{2+2b}{5}$$

$$\frac{26-3b}{7} = \frac{2+2b}{5}$$

$$5(26 - 3b) = 7(2 + 2b)$$
$$130 - 15b = 14 + 14b$$
$$130 - 130 - 15b = 14 - 130 + 14b$$
$$-15b - 14b = -116 + 14b - 14b$$
$$-29b = -116$$
$$\frac{-29b}{-29} = \frac{-116}{-29}$$

$$b = 4$$

$$7a + 3b = 26$$
$$7a + 3(4) = 26$$
$$7a + 12 = 26$$
$$7a + 12 - 12 = 26 - 12$$
$$7a = 26 - 12$$
$$7a = 14$$
$$\frac{7a}{7} = \frac{14}{7}$$

$$a = 2$$

$$(a = 2, b = 4)$$

(b)

$$6x + y = 33$$
$$4x - 2y = 14$$
$$6x + y = 33$$
$$6x + y - y = 33 - y$$
$$6x = 33 - y$$
$$\frac{6x}{6} = \frac{33-y}{6}$$
$$x = \frac{33-y}{6}$$

$$4x - 2y = 14$$
$$4x - 2y + 2y = 14 + 2y$$
$$4x = 14 + 2y$$
$$\frac{4x}{4} = \frac{14+2y}{4}$$
$$x = \frac{14+2y}{4}$$

$$\frac{33-y}{6} = \frac{14+2y}{4}$$

$$4\left(33 - y\right) = 6\left(14 + 2y\right)$$
$$132 - 4y = 84 + 12y$$
$$132 - 132 - 4y = 84 - 132 + 12y$$
$$-4y - 12y = -48 + 12y - 12y$$
$$-16y = -48$$
$$\frac{-16y}{-16} = \frac{-48}{-16}$$
$$y = 3$$

$$6x + y = 33$$
$$6x + 3 = 33$$
$$6x + 3 - 3 = 33 - 3$$
$$6x = 30$$
$$\frac{6x}{6} = \frac{30}{6}$$

$$x = 5$$

$$\left(x = 5, y = 3\right)$$

3302. Fractional Simultaneous Linear Equations

Now that you have mastered simultaneous linear equations, we will incorporate fractions in equations. When dealing with fractional equations, the procedures are the same as those in the previous paragraphs. Generally, when solving fractional equations it is just as easy to leave equations in fraction form. Examples of using both the fraction and fractions eliminated forms are given. Study the examples.

a. Fractional form. Using the fraction form you must put both equations in the same order. Find for one variable by adding the two equations and isolate by factoring. Once you have found one of the variables, put your solution back into one of the original equations and solve for the other variable. Always substitute your solutions in the original equations to check the correctness of your work.

Example:

$$\frac{4}{R_1} - \frac{3}{R_2} = 4 \qquad \text{(Original equations)}$$

$$\frac{3}{R_2} + \frac{2}{R_1} = 10$$

$$\frac{4}{R_1} - \frac{3}{R_2} = 4 \qquad \text{(Equations arranged in same order)}$$
$$\frac{2}{R_1} + \frac{3}{R_2} = 10$$

$$\frac{4}{R_1} - \frac{3}{R_2} = 4$$
$$\frac{2}{R_1} + \frac{3}{R_2} = 10 \qquad \text{(Add the two equations)}$$
$$\frac{6}{R_1} = 14$$

$$\frac{6}{R_1} = 14 \qquad \text{(Cross multiply)}$$

$$14R_1 = 6 \qquad \text{(Divide both members by 14)}$$

$$\frac{14R_1}{14} = \frac{6}{14}$$

$$R_1 = \frac{6}{14} = \frac{3}{7}$$

Substitute solutions for R_1 in original equation and solve for other variable.

$$\frac{4}{R_1} - \frac{3}{R_2} = 4$$ (Original equation)

$$\frac{4}{\frac{3}{7}} - \frac{3}{R_2} = 4$$ (Cross multiply $\frac{4}{\frac{3}{7}}$)

$$\left(\frac{4}{1} * \frac{7}{3}\right) - \frac{3}{R_2} = 4$$

$$\frac{28}{3} - \frac{3}{R_2} = 4$$ (LCD = $3R_2$)

$$\frac{28R_2 - 9}{3R_2} = 4$$ (Cross multiply)

$$28R_2 - 9 = 12R_2$$

$$28R_2 - 12R_2 = 9$$ (Isolate the literal)

$$16R_2 = 9$$

$$R_2 = \frac{9}{16}$$

Check the solution by substituting solutions for variables.

$$\frac{4}{R_1} - \frac{3}{R_2} = 4 \qquad \frac{3}{R_2} + \frac{2}{R_1} = 10$$

$$\frac{4}{\frac{3}{7}} - \frac{3}{\frac{9}{16}} = 4 \qquad \frac{3}{\frac{9}{16}} + \frac{2}{\frac{3}{7}} = 10$$

$$\left(\frac{4}{1} * \frac{7}{3}\right) - \left(\frac{3}{1} * \frac{16}{9}\right) = 4 \qquad \left(\frac{3}{1} * \frac{16}{9}\right) + \left(\frac{2}{1} * \frac{7}{3}\right) = 10$$

$$\frac{28}{3} - \frac{48}{9} = 4 \qquad \frac{48}{9} + \frac{14}{3} = 10$$

$$\frac{28}{3} - \frac{16}{3} = 4 \qquad \frac{16}{3} + \frac{14}{3} = 10$$

$$\frac{28-16}{3} = 4 \qquad \frac{16+14}{3} = 10$$

$$\frac{12}{3} = 4 \qquad \frac{30}{3} = 10$$

b. In this example we will simplify or eliminate fractions before solving. It is your choice to do this operation or not, the solution will be the same. The main point to remember is that the simpler the equation, the less likely you will make an error in your math.

Example: Solve for α and β.

$$\frac{4}{\alpha+3} = \frac{2}{3-\beta}$$ (Original equations)

$$\frac{3}{6\alpha-12} = \frac{5}{10\beta-20}$$

$$\frac{4}{\alpha+3} = \frac{2}{3-\beta}$$

$$\frac{\cancel{3}}{2*\cancel{3}(\alpha-2)} = \frac{\cancel{5}}{\cancel{5}(2\beta-4)}$$ (Reduce where possible)

$$\frac{4}{\alpha+3} = \frac{2}{3-\beta}$$

$$\frac{1}{2\alpha-4} = \frac{1}{2\beta-4}$$

$$2(\alpha+3) = 4(3-\beta)$$ (Cross multiply)

$$2\alpha-4 = 2\beta-4$$

$$2\alpha+6 = 12-4\beta$$

$$2\alpha-4 = 2\beta-4$$

$$2\alpha+6-6 = 12-6-4\beta$$ (Isolate terms)

$$2\alpha-4+4 = 2\beta-4+4$$

$$2\alpha+4\beta = 6-4\beta+4\beta$$

$$2\alpha-2\beta = 2\beta-2\beta$$

$$2\alpha+4\beta = 6$$

$$2\alpha-2\beta = 0$$ (Multiply by 2)

$$2\alpha+4\beta = 6$$ (Add)

$$4\alpha-4\beta = 0$$

$$6\alpha \qquad 6$$

$$\frac{6\alpha}{6} = \frac{6}{6}$$

$$\alpha = 1$$

Substitute the solution 1 for the variable "α" into one of the original equations and solve for "β."

$\frac{4}{\alpha+3} = \frac{2}{3-\beta}$ (Original equation)

$\frac{4}{1+3} = \frac{2}{3-\beta}$ (1 substituted for α)

$1 = \frac{2}{3-\beta}$ (Cross multiply)

$3 - \beta = 2$ (Solve for β)

$\beta = 1$

Check the solution by substituting the number 1 for the literal 'α " and 'β " into original equations.

$\frac{4}{\alpha+3} = \frac{2}{3-\beta}$ $\frac{3}{6\alpha-12} = \frac{5}{10\beta-20}$

$\frac{4}{1+3} = \frac{2}{3-1}$ $\frac{3}{6-12} = \frac{5}{10-20}$

$\frac{4}{4} = \frac{2}{2}$ $\frac{3}{-6} = \frac{5}{-10}$

$1 = 1$ $-\frac{1}{2} = -\frac{1}{2}$

Try the following challenge to reinforce your understanding of solving linear equations containing fractional equations. Substitute your solution for the literal. Check your work to ensure your solutions are correct before checking them against those given.

(Solve the fractional linear equations)

(a)

$$\frac{4}{E_1} + \frac{9}{E_2} = 5$$

$$\frac{8}{E_1} - \frac{6}{E_2} = 2$$

(b)

$$\frac{1}{2R_1+3} = \frac{8}{12Z+36}$$

$$\frac{14}{2R_1+16} = \frac{3}{Z-}$$

If your answer to the challenge is the same as below, you are correct and may continue. If your answer is incorrect, review paragraph 3302 on comparison method for solving simultaneous linear equations before continuing.

(Solve the fractional linear equations both variables)

(a)

$$\frac{4}{E_1} + \frac{9}{E_2} = 5$$
$$\frac{8}{E_1} - \frac{6}{E_2} = 2$$

$$2\left(\frac{4}{E_1}\right) + 2\left(\frac{9}{E_2}\right) = 2(5)$$

$$\frac{8}{E_1} - \frac{6}{E_2} = 2$$

$$\frac{8}{E_1} + \frac{18}{E_2} = 10$$
$$-\frac{8}{E_1} + \frac{6}{E_2} = -2$$
$$\frac{24}{E_2} = 8$$

$$E_2 * \frac{24}{E_2} = 8 * E_2$$

$$24 = 8E_2$$
$$\frac{24}{8} = \frac{8E_2}{8}$$

$$E_2 = 3$$

$$\frac{4}{E_1} + \frac{9}{3} = 3$$

$$\frac{4}{E_1} + 3 - 3 = 5 - 3$$

$$\frac{4}{E_1} = 2$$

$$E_1 * \frac{4}{E_1} = 2 * E_1$$

$$4 = 2E_1$$
$$\frac{4}{2} = \frac{2E_1}{2}$$

$$E_1 = 2$$

$$(E_1 = 2, E_2 = 3)$$

(b)

$$\frac{1}{2R_1+3} = \frac{8}{12Z+36} \qquad \frac{14}{2R_1+16} = \frac{3}{Z-1}$$
$$\frac{1}{2R_1+3} = \frac{2*2*2}{2*2*3Z+2*2*9} \qquad \frac{2*7}{2R_1+2*8} = \frac{3}{Z-1}$$
$$\frac{1}{2R_1+3} = \frac{2}{3Z+9} \qquad \frac{7}{R_1+8} = \frac{3}{Z-1}$$
$$2(2R_1+3) = 3Z+9 \qquad 3(R_1+8) = 7(Z-1)$$
$$4R_1+6 = 3Z+9 \qquad 3R_1+24 = 7Z-7$$
$$4R_1-3Z = 3 \qquad 3R_1-7Z = -31$$

$$4R_1 - 3Z = 3$$
$$3R_1 - 7Z = -31$$
$$(3)4R_1 - (3)3Z = (3)3$$
$$(4)3R_1 - (4)7Z = (4) - 31$$

$$12R_1 - 9Z = 9$$
$$-12R_1 + 28Z = 124$$
$$19Z = 133$$
$$\frac{19Z}{19} = \frac{133}{19}$$

$$Z = 7$$

$$\frac{1}{2R_1+3} = \frac{2}{21+9}$$
$$\frac{1}{2R_1+3} = \frac{2}{30}$$
$$\frac{1}{2R_1+3} = \frac{1}{15}$$
$$2R_1 + 3 = 15$$

$$2R_1 = 12$$
$$\frac{2R_1}{2} = \frac{12}{2}$$

$$R_1 = 6$$

$$(R_1 = 6, Z = 7)$$

3303. Three Unknowns in Simultaneous Linear Equations

In the preceding paragraphs, you mastered two equations with two variables or unknowns using three different methods. Two of the methods will be used to solve for three unknowns; they are elimination by addition/subtraction, and the substitution methods. Knowing how to solve linear equations with two variables (as you have already learned) and now, learning to solve equations with three variables will enable you to handle any given set of circuit parameters. If a condition should occur having more variables, you will be able to solve for three and incorporate additional unknowns if required. Study the following examples.

a. Elimination by the addition/subtraction method. When solving simultaneous linear equations that have three unknowns, you must reduce the number of variables. The first step is to choose any unknown to eliminate. In the example, "x" is eliminated. The procedure used to eliminate a variable is to select any two of the three equations and multiply either one or both by a number that will give you like coefficients. When the coefficients are the same, you can algebraically add leaving only two unknowns. The same operation must be performed with the remaining equation and one of the equations already used. You then take the two resultant equations and isolate for one variable as you did in paragraphs 3301 and 3302. Now that you have found one unknown, use the same resultant equations and solve for the other unknown. The final step is to replace the two variables in any one of the original equations and solve for the last unknown. Make sure you check the solutions by trying them in one of the other equations. Study the example.

Example:

$$\begin{aligned}2x+3y+5z&=0\\6x-2y-3z&=3\\8x-5y-6z&=1\end{aligned}$$ (Original equations) (Use any two equations)

$$\begin{aligned}2x+3y+5z&=0\\6x-2y-3z&=3\end{aligned}$$ (Multiply by 3 to eliminate x)

$$\begin{aligned}6x+9y+15z&=0\\6x-2y-3z&=3\end{aligned}$$ (Change signs and algebraically add)

$$\begin{aligned}6x+9y+15z&=0\\-6x+2y+3z&=-3\\\hline 11y+18z&=-3\end{aligned}$$ (Multiply by -1)

Use the remaining equation and one already used and do what you did before to eliminate the same variable.

$$2x + 3y + 5z = 0$$
$$8x - 5y - 6z = 1$$

$$8x + 12y + 20z = 0 \quad \text{(Multiply by 4)}$$
$$8x - 5y - 6z = 1$$

$$8x + 12y + 20z = 0$$
$$-8x + 5y + 6z = -1 \quad \text{(Change signs and algebraically add)}$$
$$17y + 26z = -1$$

Use the two new equations solve for "y."

$$11y + 18z = -3$$
$$17y + 26z = -1$$

$$13\left(11y + 18z = -3\right)$$
$$-9\left(17y + 26z = -1\right)$$

$$143y + 234z = -39$$
$$-153y - 234z = 9 \quad \text{(Algebraically add)}$$
$$-10y = -30$$

$$-10y = -30$$

$$y = \tfrac{-30}{-10}$$

$$y = 3$$

Substitute the value of "y" and solve for "z."

$$11y + 18z = -3$$

$$11(3) + 18z = -3$$

$$33 + 18z = -3$$

$$18z = -36$$

$$z = \tfrac{-36}{18}$$
$$z = -2$$

Substitute 3 for "y" and -2 for "z" values into one of the original equations and solve for "x."

$$2x + 3y + 5z = 0$$

$$2x + 3(3) + 5(-2) = 0$$

$$2x + 9 - 10 = 0$$

$$2x = 10 - 9$$

$$2x = 1$$

$$\frac{2x}{2} = \frac{1}{2}$$

$$x = \frac{1}{2}$$

Check the solution.

$$8x - 5y - 6z = 1$$

$$8\left(\frac{1}{2}\right) - 5(3) - 6(-2) = 1$$

$$\frac{8}{2} - 15 + 12 = 1$$

$$4 - 15 + 12 = 1$$

$$16 - 15 = 1$$

$$1 = 1$$

b. <u>Substitution method</u>. The first step is to choose any unknown to eliminate. In the example we will eliminate "x." The procedure used to eliminate a variable is to select any equation and solve for "x" in terms "y" and "z." You will now substitute the value "x" in the two other remaining equations. Factor these two equations and use the two new equations to solve for "y" and "z." You must take the two resultant equations and isolate for one variable as you did in the last two paragraphs. Now that you have found one unknown, use the same resultant equations and solve for the other unknown. The final step is to replace the two variables in any one of the original equations and solve for the last unknown. Make sure you check the solutions by trying them in one of the other equations. Study the example that follows.

Example:

$$2x + 3y + 5z = 0$$
$$6x - 2y - 3z = 3$$
$$8x - 5y - 6z = 1$$

(Original equations)

$$2x + 3y + 5z = 0$$
$$2x = -3y - 5z$$
$$x = \frac{-3y-5z}{2}$$

(Solve one equation for <u>x</u> in terms of <u>y</u> and <u>z</u>)

$$6x - 2y - 3z = 3$$

(Substitute $\frac{-3y-5z}{2}$ for x)

$$6\left(\frac{-3y-5z}{2}\right) - 2y - 3z = 3$$

$$3\,\cancel{6}\left(\frac{-3y-5z}{\cancel{2}}\right) - 2y - 3z = 3$$

$$-9y - 15z - 2y - 3z = 3$$
$$-11y - 18z = 3$$

(New equations)

$$8x - 5y - 6z = 1$$

(Substitute $\frac{-3y-5z}{2}$ for x)

$$8\left(\frac{-3y-5z}{2}\right) - 5y - 6z = 1$$

$$4\,\cancel{8}\left(\frac{-3y-5z}{\cancel{2}}\right) - 5y - 6z = 1$$

$$-12y - 20z - 5y - 6z = 1$$
$$-17y - 26z = 1$$

(New equation)

$$-11y - 18z = 3$$
$$-17y - 26z = 1$$

(Solve new equations for x and y)

$$17(-11y - 18z = 3)$$
$$-11(-17y - 26z = 1)$$

(Eliminate y)

$$-187y - 306z = 51$$
$$187 + 286z = -11$$
$$-20z = 40$$

(Add)

$$-20z = 40$$
$$\frac{-20z}{-20} = \frac{40}{-20}$$
$$z = -2$$

(Substitute - 2 for *z* and solve for *y*)

$$-11y - 18z = 3$$
$$-11y - 18(-2) = 3$$
$$-11y + 36 = 3$$
$$-11y + 36 - 36 = 3 - 36$$
$$-11y = -33$$
$$\frac{-11y}{-11} = \frac{-33}{-11}$$
$$y = 3$$

(Substitute for *z* and *y*, and solve for *x*)

$$2x + 3y + 5z = 0$$
$$2x + 3(3) + 5(-2) = 0$$
$$2x + 9 - 10 = 0$$
$$2x + 9 - 9 - 10 + 10 = 10 - 9$$
$$2x = 1$$
$$\frac{2x}{2} = \frac{1}{2}$$
$$x = \frac{1}{2}$$

As you can see the solution is the same $\left(x = \frac{1}{2}, y = 3, z = -2\right)$ using the substitution method as it is with the addition/subtraction method.

Try the following challenge to reinforce your knowledge. You may use either the addition/ subtraction or substitution method to solve this challenge. Substitute your solution for the literal to ensure your solutions are correct before checking them against those given.

(Solve the linear equations using the addition/subtraction, or substitution methods)

(a)

$$X_L + 2X_C + R = 9$$
$$2X_L + X_C + R = 16$$
$$X_L + X_C + 2R = 3$$

(b)

$$E_1 - E_2 + E_3 = 2$$
$$E_1 + E_2 + E_3 = 6$$
$$E_1 + E_2 - E_3 = 0$$

If your answers to equations in the challenge are the same as follows, you are correct; you may continue. If your answers are incorrect, review paragraph 3303 on substitution method for solving simultaneous linear equations before continuing.

(Solutions to linear equations)

(a)

$$X_L + 2X_C + R = 9$$
$$2X_L + X_C + R = 16$$
$$X_L + X_C + 2R = 3$$

$$2(X_L + 2X_C + R = 9)$$
$$\underline{-1(2X_L + X_C + R = 16)}$$
$$3X_C + R = 2$$

$$2X_L + X_C + R = 16$$
$$\underline{-2(X_L + X_C + 2R = 3)}$$
$$-X_C - 3R = 10$$

$$3X_C + R = 2$$
$$\underline{3(-X_C - 3R = 10)}$$
$$-8R = 32$$
$$R = -4$$

$$3X_C + R = 2$$
$$3X_C + (-4) = 2$$
$$3X_C = 6$$
$$X_C = 2$$

$$X_L + 2X_C + R = 9$$
$$X_L + 2(2) + (-4) = 9$$
$$X_L + 4 - 4 = 9$$
$$X_L = 9$$

$$(X_L = 9, X_C = 2, R = -4)$$

(b)

$$E_1 - E_2 + E_3 = 2$$
$$E_1 + E_2 + E_3 = 6$$
$$E_1 + E_2 - E_3 = 0$$

$$E_1 - E_2 + E_3 = 2$$
$$E_1 - E_2 + E_2 + E_3 - E_3 = 2 + E_2 - E_3$$
$$E_1 = 2 + E_2 - E_3$$

$$E_1 + E_2 + E_3 = 6$$
$$(2 + E_2 - E_3) + E_2 + E_3 = 6$$
$$2 + 2E_2 = 6$$
$$2 - 2 + 2E_2 = 6 - 2$$
$$2E_2 = 4$$
$$E_2 = 2$$

$$(2 + E_2 - E_3) + E_2 - E_3 = 0$$
$$2 + 2E_2 - 2E_3 = 0$$
$$2 - 2 + 2E_2 - 2E_3 = 0 - 2$$
$$2E_2 - 2E_3 = -2$$

$$2E_2 \qquad = 4$$
$$\underline{-1(2E_2 - 2E_3) = -2}$$
$$2E_3 = 6$$
$$E_3 = 3$$

$$E_1 - E_2 + E_3 = 2$$
$$E_1 - (2) + (3) = 2$$
$$E_1 + 1 = 2$$
$$E_1 + 1 - 1 = 2 - 1$$
$$E_1 = 1$$

$$(E_1 = 1, E_2 = 2, E_3 = 3)$$

3304. Solve Determinant Matrix for Simultaneous Linear Equations

When working in electronics, you will often have to consider many conditions to solve a problem or provide a repair. You have worked with simultaneous linear equations of the second order, two variables and two equations, and also with the third order, three equations with three variables. Up to this point, you have learned and used three different methods for solving simultaneous linear equations. As you can see, the more unknowns a set of equations has, the more tedious it is to find the solution and the easier it is to make a mistake. You may find that determinants are an easier way to work with equations of the third order and higher. Just think of determinant math as one more tool in your math toolbox. You will now learn to solve this type of equation. The determinant matrix is a mechanical method derived from the comparison method. The first example shows you the procedures used when solving a linear equation of the second order using the comparison method. This allows you to compare it to the matrix and understand how the determinant matrix works and how it differs from the comparison method.

Example:

$$3b + 3a = 12$$
$$5a - 2b = 6$$

(Solve for the literal "a" in both equations)

$$3a + 3b = 12 \qquad\qquad 5a - 2b = 6$$
$$3a = 12 - 3b \qquad\qquad 5a = 6 + 2b$$

$$a = \frac{12-3b}{3} \qquad a = a \qquad a = \frac{6+2b}{5}$$

$$\frac{12-3b}{3} = \frac{6+2b}{5}$$

$$\frac{5(12-3b)}{15} = \frac{3(6+2b)}{15} \quad \text{(LCD = 15)}$$

$$\frac{15}{1} * \frac{5(12-3b)}{15} = \frac{3(6+2b)}{15} * \frac{15}{1}$$

$$5(12 - 3b) = 3(6 + 2b)$$

$$60 - 15b = 18 + 6b$$

$$-15b - 6b = 18 - 60$$

$$-21b = -42$$

$$\frac{-21b}{-21} = \frac{-42}{-42}$$

$$b = 2$$

Note: Notice that the denominators are the same and that the literal "a" and its coefficient are not present in the equation. The determinant matrix derives its method from these factors. The following steps explain how to set up a matrix.

The first step in setting up the determinant matrix is to place the equations in the same order, with numerical value in front of the literal.

Note: The importance of this step will become clear as you solve equations using a matrix. The original equation is shown below and the order changed.

Original equation	Same order
$3b + 3a = 12$	$3a + 3b = 12$
$5a - 2b = 6$	$5a - 2b = 6$

Two by two matrix. You need to know how to set up a two by two matrix. The name comes from the number of horizontal rows and vertical columns in the matrix. Two equations with two variables makes up a two by two matrix. Look at the example below with the coefficients and signs of the literal for "a" and "b" and how they are applied.

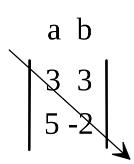
Example:

$$\begin{matrix} a & b \end{matrix}$$
$$\begin{vmatrix} 3 & 3 \\ 5 & -2 \end{vmatrix}$$

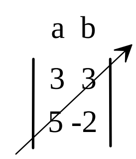
Note. Observe that the coefficients from each equation occupy the same row and maintain their signs. It does not matter which equation comes first as long as the order is maintained throughout the process.

The result forms the denominator of the matrix. You will multiply diagonally from top left to bottom right and bottom left to top right. Diagonal multiplication in determinants derives its sign from the direction of the multiplication. Positive multiplication is from upper left to lower right with the term in parenthesis preceded by a positive sign. Negative multiplication is from lower left to upper right with the term in parenthesis preceded by a negative sign. There is no algebraic significance to this procedure. Remember that determinant math is purely a mechanical process. Study the examples below to see how the signs apply to diagonal multiplication.

Examples:



Positive Direction



on

The denominator for the determinant is $[(3)(-2)-(5)(3)]=-6-15=-21$

<u>Note</u>: The positive direction when multiplying a matrix is diagonally left to right and the negative direction when multiplying a matrix is diagonally right to left. It does not matter which is done first because you are algebraically adding. We will do the positive direction first so there is no confusion.

Try the following challenge to reinforce your knowledge. Check your work to ensure your solutions are correct before checking them against those given.

(Solve the matrixes and perform algebraic addition)

(a)

$$\begin{vmatrix} 3 & -7 \\ 4 & 2 \end{vmatrix}$$

(b)

$$\begin{vmatrix} 2 & -8 \\ -3 & -5 \end{vmatrix}$$

(c)

$$\begin{vmatrix} 4 & -7 \\ 2 & -2 \end{vmatrix}$$

(d)

$$\begin{vmatrix} 4 & 9 \\ 7 & -6 \end{vmatrix}$$

If your answer to the challenge is the same as follows; continue. If your answer is incorrect, review paragraph 3304 on determinant matrix for solving simultaneous linear equations before continuing.

(Solution to matrixes)

(a)

$$\begin{vmatrix} 3 & -7 \\ 4 & 2 \end{vmatrix} = ((3)\ (2) - (4)\ (-7)) = 6 + 28 = 34$$

(b)

$$\begin{vmatrix} 2 & -8 \\ -3 & -5 \end{vmatrix} = ((2)\ (-5) - (-3)\ (-8)) = -10 -24 = -34$$

(c)

$$\begin{vmatrix} 4 & -7 \\ 2 & -2 \end{vmatrix} = ((4)\ (-2) - (2)\ (-7)) = -8 +14 = 6$$

(d)

$$\begin{vmatrix} 4 & 9 \\ 7 & -6 \end{vmatrix} = ((4)\ (-6) - (7)\ (9)) = -24 - 63 = -87$$

Now that you know the procedure for finding the determinant denominator, you are ready to set up the matrix for the numerator. To solve for the literal "a" on the top of the matrix, place the sign and numerical coefficient for the literal "b" and replace the literal "a" numerical coefficients with the independent values. You will do the opposite operation when you are solving for the literal "b." Remember, the same rules apply to diagonal multiplication in the numerator as you applied it with the denominator. Study the two examples shown explaining the procedures.

Examples:

$$3a + 3b = 12$$
$$5a - 2b = 6$$

$$a = \frac{\begin{vmatrix} 12 & 3 \\ 6 & -2 \end{vmatrix}}{\begin{vmatrix} 3 & 3 \\ 5 & -2 \end{vmatrix}} = \frac{((12)\ (-2)) - ((6)\ (3))}{((3)\ (-2)) - ((5)\ (3))}$$

$$a = \frac{(-24) - (18)}{(-6) - (15)}$$

$$a = \frac{-24 - 18}{-6 - 15}$$

$$a = \frac{-42}{-21}$$

$$a = 2$$

$$b = \frac{\begin{vmatrix} 3 & 12 \\ 5 & 6 \end{vmatrix}}{-21} = \frac{((3)\ (6)) - ((5)\ (12))}{-21}$$

$$b = \frac{(18) - (60)}{-21}$$

$$b = \frac{18 - 60}{-21}$$

$$b = \frac{-42}{-21}$$

$$b = 2$$

<u>Note</u>: Since the denominator is a product of the coefficients of both literal and does not change whether you are solving for "a" or "b," the denominator does not change as shown in the second part of the example.

Try the following challenge. Check your work to ensure your solutions are correct before checking your solutions against those given.

(Solve the following equations using the determinant matrix)

(a)

$$\pi - 8\omega = 0$$
$$\pi + \omega = 45$$

(b)

$$3z - 2R = 7$$
$$z + 2R = 5$$

If your answers to the equations in the challenge are the same as the following, you are correct. Continue. If your answers are incorrect, review paragraph 3304 on determinant matrix for solving simultaneous linear equations before continuing.

Note: Review the following solutions using the determinant matrix to solve simultaneous linear equations. Compare your solutions with the following.

(a)

$$\pi - 8\omega = 0$$
$$\pi + \omega = 45$$

$$\pi = \frac{\begin{vmatrix} 0 & -8 \\ 45 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & -8 \\ 1 & 1 \end{vmatrix}} = \frac{((0)(1)) - ((45)(-8))}{((1)(1)) - ((1)(-8))}$$

(columns: π, ω)

$$\pi = \frac{0+360}{1+8}$$
$$\pi = \frac{360}{9}$$
$$\pi = 40$$

$$\omega = \frac{\begin{vmatrix} 1 & 0 \\ 1 & 45 \end{vmatrix}}{9} = \frac{((1)(45)) - ((1)(0))}{9}$$

(columns: π, ω)

$$\omega = \frac{45-0}{9}$$
$$\omega = \frac{45}{9}$$
$$\omega = 5$$

$$(\pi = 40,\ \omega = 5)$$

(b)

$$3z - 2R = 7$$
$$z + 2R = 5$$

$$z = \frac{\begin{vmatrix} 7 & -2 \\ 5 & 2 \end{vmatrix}}{\begin{vmatrix} 3 & -2 \\ 1 & 2 \end{vmatrix}} = \frac{((7)(2)) - ((5)(-2))}{((3)(2)) - ((1)(-2))}$$

(columns: z, R)

$$z = \frac{14+10}{6+2}$$
$$z = \frac{24}{8}$$
$$z = 3$$

$$R = \frac{\begin{vmatrix} 3 & 7 \\ 1 & 5 \end{vmatrix}}{8} = \frac{((3)(5)) - ((1)(5))}{8}$$

(columns: z, R)

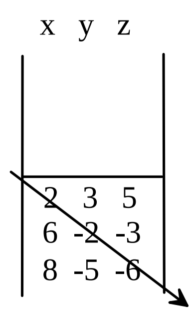
$$R = \frac{15-7}{8}$$
$$R = \frac{8}{8}$$
$$R = 1$$

$$(z = 3,\ R = 1)$$

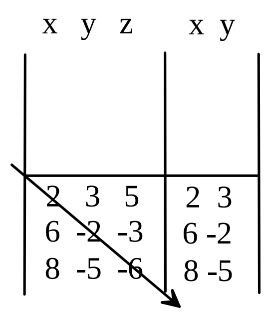
You have mastered determinants to the second order; you are ready for determinants of the third order. You must learn to set up the matrix for the third order. You must apply the principles that you learned for setting up matrices earlier. These principles will work for determinants of any order. The first step is exactly the same as you did for the second order. Place the equations in the same order, with numerical value in front of the literal. Set up a three by three matrix as you set up your two by two matrix, except this time, the matrix will have three primary columns with three rows. The first two columns will have to be on the right hand side of the matrix to allow you to have the right number of products from multiplication. As you examine the example, you will see that repeating the first two columns to the right of the matrix is done for simplicity reasons. We will demonstrate that the positive direction and the negative direction works the same way.

Example: Denominator

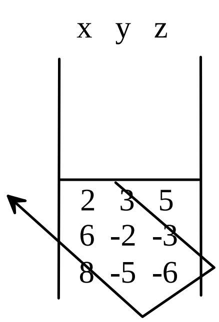
$$\begin{aligned} 2x + 3y + 5z &= 0 \\ 6x - 2y - 3z &= 3 \\ 8x - 5y - 6z &= 1 \end{aligned}$$



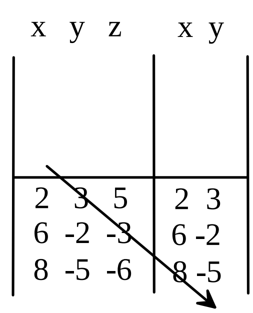
2 * -2 * -6 = 24



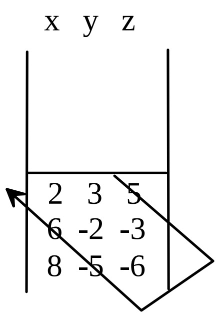
2 * -2 * -6 = 24



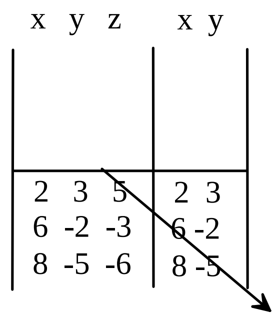
3 * -3 * 8 = -72



3*-3*8=-72

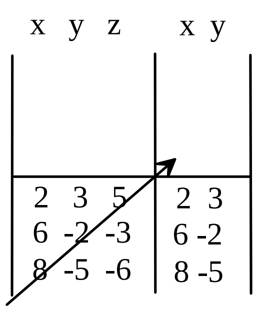


5 * -5 * 6 = -150

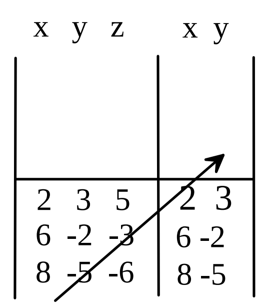


5 * 6 * -5 = -150

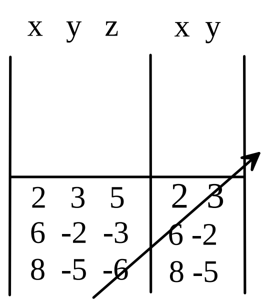
You will less likely make a mistake if you copy the columns to right of the matrix. The applications of this principle will ensure the correctness of your work if you are required to set up a determinant matrix of this or higher order. Complete the process of solving the equation by multiplying the numerals in the denominator in the negative direction using the matrix with the columns added.



- (8 * -2 * 5) = - (- 80)



- (-5 * - 3 * 2) = - (30)



- (-6*6*3) = - (- 108)

<u>Note</u>: Just as yo ... e the values for the denominator for a three by three.

$$\overline{(24)+(-72)+(-150)-(-80)-(30)-(-108)}$$

Now fill in the entire matrix and solve for the literal "x." To solve for "x," on the top of the matrix place the sign and numerical coefficient for the literal "y" and "z," and replace the literal "x" numerical coefficients with the independent values. You will do the same operation when solving for the literal "y." Remember, the same rules apply to diagonal multiplication in the numerator as you did with the denominator. Study the example given showing a completed matrix using the original equations.

Example:

$$2x + 3y + 5z = 0$$
$$6x - 2y - 3z = 3$$
$$8x - 5y - 6z = 1$$

$$x = \frac{\begin{array}{ccc|cc} x & y & z & x & y \\ 3 & -2 & -3 & 3 & -2 \\ 1 & 5 & -6 & 1 & -5 \end{array}}{\begin{array}{ccc|cc} 2 & 3 & 5 & 2 & 3 \\ 6 & -2 & -3 & 6 & -2 \\ 8 & -5 & -6 & 8 & -5 \end{array}} = \frac{(0)+(-9)+(-75)-(-10)-(0)-(-10)}{(24)+(-72)+(-150)-(-80)-(30)-(-108)}$$

$$x = \frac{-9-75+10+54}{24-72-150+80-30+108}$$

$$x = \frac{-20}{-40}$$

$$x = \frac{1}{2}$$

<u>Note</u>: Denominator does not change.

$$y = \frac{\begin{array}{ccc|cc} x & y & z & x & y \\ 2 & 0 & 5 & 2 & 0 \\ 6 & 3 & -3 & 6 & 3 \\ 8 & 1 & -6 & 8 & 1 \end{array}}{-40} = \frac{(-36)+(0)+(30)-(120)-(-6)-(0)}{-40}$$

$$y = -\frac{-36+30-120+6}{-40}$$

$$y = \frac{-120}{-40}$$

$$y = 3$$

To solve for the last literal "z," perform determinant matrix by replacing only numerical coefficients for "z." The easiest solution is to use one of the original equations and substitute the known values for the literal "x" and "y" and solve for "z." In the equation below, the solutions are substituted for the literal "x" and "y" and solved for "z." When you check your work, you should use one of the other equations to check your solution.

$$2\left(\tfrac{1}{2}\right) + 3(3) + 5z = 0$$

$$\begin{aligned} 1 + 9 + 5z &= 0 \\ 10 - 10 + 5z &= 0 - 10 \\ 5z &= -10 \end{aligned}$$

$$\tfrac{5z}{5} = \tfrac{-10}{5}$$

$$z = -2$$

Solution: $\left(x = \tfrac{1}{2}, y = 3, z = -2\right)$

Try the following challenge to give you more practice and to reinforce what you have already learned. Check your work to ensure your solutions are correct before checking them against those given.

(Solve using the determinant matrix the simultaneous linear equations)

(a)

$$
\begin{aligned}
2x - 4y + 5z &= 25 \\
3x - 6y - 2z &= -10 \\
7x - 2y - 4z &= 16
\end{aligned}
$$

(Continued)

(Solve using the determinant matrix the simultaneous linear equations)

(b)

$$\begin{aligned} a - 6b + 8c &= -1 \\ 8a - 3b + 2c &= 20 \\ 7a - b - 4c &= 15 \end{aligned}$$

(Solution using the determinant matrix the simultaneous linear equations)

(a)

$$2x - 4y + 5z = 25$$
$$3x - 6y - 2z = -10$$
$$7x - 2y - 4z = 16$$

$$x = \frac{\begin{vmatrix} 25 & -4 & 5 \\ -10 & -6 & -2 \\ 16 & -2 & -4 \end{vmatrix}\begin{matrix} 25 & -4 \\ -10 & -6 \\ 16 & -2 \end{matrix}}{\begin{vmatrix} 2 & -4 & 5 \\ 3 & -6 & -2 \\ 7 & -2 & -4 \end{vmatrix}\begin{matrix} 2 & -4 \\ 3 & -6 \\ 7 & -2 \end{matrix}} = \frac{(600)+(128)+(100)-(-480)-(100)-(-160)}{(48)+(56)+(-30)-(-210)-(8)-(48)}$$

(columns labelled x y z | x y)

$$x = \tfrac{1368}{228} = 6$$

$$x = 6$$

$$y = \frac{\begin{vmatrix} 2 & 25 & 5 \\ 3 & -10 & -2 \\ 7 & 16 & -4 \end{vmatrix}\begin{matrix} 2 & 25 \\ 3 & -10 \\ 7 & 16 \end{matrix}}{228} = \frac{(80)+(-350)+(240)-(-350)-(-64)-(-300)}{228}$$

(columns labelled x y z | x y)

$$y = \tfrac{684}{228} = 3$$

$$y = 3$$

$$2x - 4y + 5z = 25$$
$$2(6) - 4(3) + 5z = 25$$
$$12 - 12 + 5z = 25$$
$$5z = 25$$
$$\tfrac{5z}{5} = \tfrac{25}{5}$$
$$z = 5$$

$(x = 6,\ y = 3,\ z = 5)$

(Solution using the determinant matrix the simultaneous linear equations)

(b)

$$a - 6b + 8c = -1$$

$$8a - 3b + 2c = 20$$

$$7a - b - 4c = 15$$

$$a = \frac{\left|\begin{array}{ccc|cc} \text{a} & \text{b} & \text{c} & \text{a} & \text{b} \\ -1 & -6 & 8 & -1 & 6 \\ 20 & -3 & 2 & 20 & -3 \\ 15 & -1 & -4 & 15 & -1 \end{array}\right|}{\left|\begin{array}{ccc|cc} 1 & -6 & 8 & 1 & -6 \\ 8 & -3 & 2 & 8 & -3 \\ 7 & -1 & -4 & 7 & -1 \end{array}\right|} = \frac{(-12)+(-180)+(-160)-(-360)-(2)-(480)}{(12)+(-84)+(-64)-(-168)-(-2)-(192)}$$

$$a = \frac{-474}{-158} = 3$$

$$b = \frac{\left|\begin{array}{ccc|cc} \text{a} & \text{b} & \text{c} & \text{a} & \text{b} \\ 1 & -1 & 8 & 1 & -1 \\ 8 & 20 & 2 & 8 & 20 \\ 7 & 15 & -4 & 7 & 15 \end{array}\right|}{-158} = \frac{(-80)+(-14)+(960)-(1120)-(30)-(32)}{-158}$$

$$b = \frac{-316}{-158} = 2$$

$$b = 2$$

$$a - 6b + 8c = -1$$

$$3 - 6(2) + 8c = -1$$

$$3 - 12 + 8c = -1$$

$$-9 + 9 + 8c = -1 + 9$$

$$\frac{8c}{8} = \frac{8}{8}$$

$$c = 1$$

$$(a = 3,\ b = 2,\ c = 1)$$

Lesson Summary. You have completed the last lesson of this course. In this lesson, you learned to solve simultaneous linear equations containing two and three unknowns. You solved simultaneous linear equations using addition/subtraction, substitution, and comparison methods. You also learned to set up and solve multiple equations having multiple unknowns using the determinant matrix. You will be able to apply your acquired skills and knowledge on electronic formulas as required. Complete the lesson exercise.

Lesson 3 Exercise: Complete items 1 through 14 by performing the action required. Check your responses against those listed at the end of this lesson.

1. Solve for the literal numbers (x, y) using the addition/subtraction method.

$$5x - 7y = 1$$
$$3x + 3y = 15$$

2. Solve for the literal numbers (a, b) using the addition/subtraction method.

$$6a - 3b = 21$$
$$3a + 7b = 36$$

3. Solve for the literal numbers (x, y) using the substitution method.

$$\begin{aligned} 7x - 5y &= -11 \\ 2x + 5y &= \ \ 29 \end{aligned}$$

4. Solve for the literal numbers (a, b) using the substitution method.

$$7a - 5b = 13$$
$$5a + 3b = 29$$

5. Solve for the literal numbers (x, y) using the comparison method.

$$7x - 3y = 26$$
$$9x + 2y = 51$$

6. Solve for the literal numbers (a, b) using the substitution method.

$$5a + 2b = 18$$
$$7a - 3b = 2$$

7. Solve the fractional linear equation.

$$\frac{14}{3R_1+16} = \frac{7}{2Z+3}$$

$$\frac{1}{2R_1+3} = \frac{8}{12Z+36}$$

8. Solve the fractional linear equation.

$$\frac{4}{E_1} + \frac{6}{E_2} = 13$$

$$\frac{2}{E_1} + \frac{4}{E_2} = \frac{1}{2}$$

9. Solve the linear equation using the addition/subtraction or substitution method.

$$\begin{aligned} 2E_1 + 2E_2 + 2E_3 &= 12 \\ E_1 - E_2 + E_3 &= 2 \\ E_1 + E_2 - E_3 &= 0 \end{aligned}$$

10. Solve the linear equation using the addition/subtraction or substitution method.

$$\begin{aligned} 2X_L + X_C + R &= 16 \\ X_L + 2X_C + R &= 9 \\ X_L + X_C + 2R &= 3 \end{aligned}$$

11. Solve the equation using the determinant matrix.

$$\begin{aligned} 3z + 6R &= 15 \\ 3z - 2R &= 7 \end{aligned}$$

12. Solve the equation using the determinant matrix.

$$\begin{aligned} 2\pi + 3\omega &= 95 \\ \pi - 8\omega &= 0 \end{aligned}$$

13. Solve the equation using the determinant matrix.

$$\begin{aligned} a - 6b + 8c &= -1 \\ 7a + 3b - 4c &= 23 \\ 8a - 3b + 2c &= 20 \end{aligned}$$

14. Solve the equation using the determinant matrix.

$$7x - 2y - 4z = 16$$
$$3x - 6y - 2z = -10$$
$$2x + 4y + 5z = 49$$

Check your answers with those on the next page. Review those items that you missed before continuing to the study unit exercise.

Lesson 3 Exercise Solutions

		Reference
1.	$x = 3, y = 2$	3301
2.	$a = 5, b = 3$	3301
3.	$x = 2, y = 5$	3301
4.	$a = 4, b = 3$	3301
5.	$x = 5, y = 3$	3301
6.	$a = 2, b = 4$	3301
7.	$R_1 = 6, z = 7$	3302
8.	$E_1 = \frac{4}{49}, E_2 = \frac{-1}{6}$	3302
9.	$E_1 = 1, E_2 = 2, E_3 = 3$	3303
10.	$X_L = 9, X_c = 2, R = -4$	3303
11.	$R = 1, z = 3$	3304
12.	$\pi = 40, \omega = 5$	3304
13.	$a = 3, b = 2, c = 1$	3304
14.	$x = 6, y = 3, z = 5$	3304

Unit Summary. In this study unit, you learned the procedures for manipulating equations to solve for variables. You learned to apply the seven axioms to literal equations. You are able to apply problem solving procedures to equations involving ratio and proportions. You learned to solve fractional equations by removing the fractions or by factoring with the LCD. You learned to solve simultaneous linear equations using the elimination by addition/subtraction, substitution, and comparison methods. Finally, you mastered the determinant matrix. This course provided you with the mathematical skills necessary to work with electronic formulas and will help you complete higher level mathematical problems.

Study Unit 3 Exercise: Complete items 1 through 20 by performing the action required. Check your responses against those listed at the end of this study unit exercise.

1. $\frac{2a+4}{3} - \frac{a+1}{2} = \frac{2}{3}$ Solve for a.

a. 1

b. -3

c. -1

d. -1.5

2. $R_x = \frac{1}{\omega_1 C} - R_n$ Solve for ω_1.

a. $\frac{1}{C(R_x+R_n)}$

b. $\frac{C}{R_x+R_n}$

c. $\frac{R_x+R_n}{C}$

d. $C(R_x + R_n)$

3. $19 - 5a(4a+1) = 40 - 10a(2a-1)$ Solve for a.

a. $-\frac{5}{7}$

b. $-\frac{7}{5}$

c. $-\frac{6}{7}$

d. $-\frac{6}{5}$

4. $LZ = \frac{2y}{2W+y}$ Solve for y.

a. $\frac{W}{LZ}$

b. $\frac{2-LZ}{2LWZ}$

c. $\frac{LWZ}{2}$

d. $\frac{2LWZ}{2-LZ}$

5. $\frac{X^2+3X}{X} \div \frac{X+3}{X} = \frac{X+2}{3} + \frac{2X-3}{4}$ Solve for X.

a. 8.5

b. -3.2

c. -1.5

d. $-.5$

6. $(.7a-.7)(.2+a)=(1-1.4a)(.1-.5a)$

Solve for a.

a. 4 | c. .02

b. 3 | d. .3

7. $\frac{1.3x-1.5}{30}=\frac{.4x+.3}{5}$ Solve for x.

a. –3 | c. –2

b. –5 | d. 3

8. $R_o=R_c+\frac{R_B}{\beta}$ Solve for β.

a. $\frac{R_o-R_c}{R_B}$ | c. $\frac{R_B}{R_o-R_c}$

b. $\frac{R_o-R_B}{R_c}$ | d. $\frac{R_c}{R_o-R_B}$

9. $\omega=\frac{\beta^2 AL}{2\mu_o}$ Solve for β.

a. $AL\sqrt{2\mu_o\omega}$ | c. $\sqrt{\frac{AL}{2\mu_o\omega}}$

b. $\sqrt{\frac{2\mu_o\omega}{AL}}$ | d. $2\mu_o\omega\sqrt{AL}$

10. $2x+3y=16$
$5x+2y=29$ Solve for x.

a. 5 | c. 6

b. 3 | d. 4

11. $3a + 4b + c = 15.5$
$2a - 2b - 3c = 1.5$
$4a + b - 2c = 12.5$

Solve for a.

a. –3

b. 2.5

c. 3

d. 3.5

12. See item 11

Solve for c.

a. 5

b. .5

c. .2

d. 1.5

13. $x = \frac{G(A-a)}{Z}$

Solve for A.

a. $a - \frac{ZX}{G}$

b. $\frac{G}{ZX} + a$

c. $\frac{XG}{Z} - a$

d. $\frac{ZX}{G} + a$

14. $2A - 3B + 5C = 21$
$A - 2B - C = -4$
$4A - B + 3c = 19$

Solve for B.

a. –1

b. 1

c. 3

d. 2

15. See item 14

Solve for C.

a. 2

b. 5

c. 3

d. 4

16. $\frac{2x}{3} + \frac{x}{4} - \frac{3x}{6} = \frac{x}{2} + \frac{3x}{4} - \frac{1}{3}$ Solve for x.

a. $\frac{2}{5}$

b. $\frac{2}{3}$

c. $\frac{3}{4}$

d. $\frac{1}{4}$

17. Solve for I_3.

$$\begin{aligned} 2I_1 - 4I_2 + 6I_3 &= -8 \\ 10I_1 - 5I_2 + 3I_3 &= 8 \\ I_1 + 2I_2 + 4I_3 &= 17 \end{aligned}$$

a. 3

b. 2

c. 1

d. 5

18. See item 17 Solve for I_1.

a. 2

b. 3

c. 4

d. 1

19. $\frac{a-7}{a+2} - \frac{6}{a+3} = \frac{a^2-a-42}{a^2+5a+6}$ Solve for a.

a. a

b. 6

c. 2

d. 1

20. $\frac{10}{W} - 3 = \frac{2-W}{W}$ Solve for W.

a. 4

b. 2

c. $\frac{1}{2}$

d. 3

Check your answers with those on the next page. Review those items that you missed before you take your course examination.

Study Unit 3 Exercise Solutions

		Reference
1.	c.	3201
2.	a.	3104
3.	b.	3104
4.	d.	3103
5.	d.	3202
6.	b.	3202
7.	a.	3202
8.	c.	3103
9.	b.	3104
10.	a.	3303
11.	c.	3304
12.	b.	3304
13.	d.	3104
14.	b.	3304
15.	d.	3304
16.	a.	3201
17.	c.	3304
18.	b.	3304
19.	d.	3104
20.	a.	3201

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